Wm S. N. 1786

THE
Complete Meaſurer.

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OR, THE
Whole Art of Meaſuring.

In Two Parts.

PART I.

Teaching Decimal Arithmetic, with the Extraction of the Square and Cube-Roots.

Alſo the Multiplication of Feet and Inches, commonly called *Croſs-Multiplication.*

PART II.

Teaching to meaſure all Sorts of Superficies and Solids, by *Decimals*, by *Croſs-Multiplication*, and by *Scale* and *Compaſſes*: Alſo the Works of ſeveral Artificers relating to Building; and the meaſuring of Board and Timber; ſhewing the common Errors. And ſome Practical Queſtions.

The Seventh Edition, much Enlarged.
To which is divided an Appendix, 1. Of *Gaging.* 2. Of *Land Meaſuring.*

Very uſeful for all Tradeſmen, eſpecially Carpenters, Bricklayers, Plaiſterers, Painters, Joyners, Glaſiers, Maſons, *&c.*

By WILLIAM HAWNEY, *Philomath.*

Recommended by the Rev. Dr. John Harris. F.R.S.

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Jas. Steele Nicholson

24th June 1842

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PREFACE.

HAVING peruſed ſeveral Books concerning the Menſuration of Superficies and Solids, and the Works of Artificers relating to Building; but not finding any one Book ſo perfect, as to give any tolerable ſatisfaction to a Learner; and I having practiſed and taught Meaſuring for ſeveral Years, and thereby gained Experience and Knowledge in that Art, having learned ſome Things from one Author, and ſome Things from another, I began to think of digeſting my Thoughts into ſome ſuch Method as might give a Learner full Satisfaction, without being at the Charge of buying ſo many Books; and being importuned thereunto by ſome Friends, I fell to work, and at laſt brought them to that Perfection you here find in the following WORK.

1. As to the DECIMAL ARITHMETIC, I have been as brief as the Matter would well bear to make it plain.

2. As to the multiplying of Feet and Inches, commonly called CROSS-MULTIPLICATION, my Method differs from that which is uſually taught in other Authors, as being (I think) much ſhorter and plainer.

3. In meaſuring of Superficies and Solids, I have given the Demonſtration of the Rules, which I thought might be very acceptable to the Ingenious; for indeed I always look upon the writing of a Rule without a Demonſtration, (in any part of the Mathematicks) to be but lame and defective; and for want of knowing the Reaſon of the Rule, a Learner may commit great Errors: Beſides, when a Learner knows the Reaſon of the Rules, he may retain them better in his Memory. The Rule for meaſuring a Priſmoid and Cylindroid, I had out of Mr. *Everard's Art of Gaging*; but the Reaſon he does not ſhew, neither have I found it in any other Author; but that the Method is true, I have endeavoured to make plain.

The Demonſtrations of the Rules for finding the Area of an Ellipſis and Parabola; alſo the Demonſtration of the Rules for finding the ſolid Content of the Fruſtum of a Cone and Pyramid, the Solidity of a Globe, of a Spheroid, a Parabolick Conoid, and of a Parabolick Spindle, and their Fruſtums, I had from the ingenious Mr. *Ward's Young Mathe-*

Mathematician's Guide ; where the curious and ingenious Reader may ſee many other Demonſtrations algebraically performed : I have alſo demonſtrated the Rule for finding the Solidity of a Globe, out of *Pardie's Elements of Geometry*, (Book Vth. Art. the 33d.) publiſhed in *Engliſh*, with many Additions, by the Reverend Dr. *Harris*, F. R. S. and the ſame is alſo done out of *Sturmius's Matheſis Enucleata* ; ſo that the ingenious Reader may uſe which of thoſe Ways he likes beſt.

The Scale ſuppoſed to be uſed in all the Operations is the Line of Numbers, commonly called *Gunter's Line*, which is upon the ordinary two Feet or eighteen Inch Rules, commonly uſed by the Carpenters, Maſons, *&c.* becauſe I thought it needleſs, as well as impertinent, to write the Uſe of Sliding-Rules, or any other particular Scales, they being ſufficiently treated of ſeveral Authors, *viz.* by the above-named Mr. *Everard*, in his *Art of Gaging*, above-mentioned, where you have the Uſe of a Sliding-Rule in Arithmetic, Geometry, in meaſuring of Superficies and Solids, Gaging, *&c.* Likewiſe Mr. *Hunt* has wrote largely of the Uſe of his Sliding-Rule, in a Arithmetic, Geometry, Trigonometry, Gaging, Dyalling, *&c.* There are ſeveral others who have explained the Uſe of their own Rules ; ſo that the more curious Readers may find full Satiſfaction in thoſe Authors.

One Thing I have omitted in the Book, which I think may not be very improperly inſerted in this Place; that is, how to find a Number upon the Line. If the Number you will find conſiſts only of Units, then the Figures upon the Line repreſent the Number ſought: Thus, if the Number be 1, 2, 3, *&c.* then 1, 2, 3, *&c.* upon the Line, repreſent the Number ſought; but if the Number conſiſts of two Figures, that is, of Units and Tens, then the Figure upon the Rule ſtands for Tens, and the larger Diviſions ſtand for Units: Thus, if 34 were to be found upon the Line, the Figure 3 upon the Line is 30, and 4 of the large Diviſions (counted forward) is the Point repreſenting 34; and if 340 were to be found, it will be at the ſame Point upon the Line; and if 304 were to be found, then the 3 upon the Line is 300, and 4 of the ſmaller Diviſions (counted forward) is the Point repreſenting 304: If the Number conſiſts of four Places, or Thouſands, then the Figure upon the Line ſtands for Thouſands, and the larger Diviſions are Hundreds, the leſſer Diviſions are Tens, and the tenth Parts of thoſe leſſer Diviſions are Units; thus, if 2735 were to be found, then the 2 is 2000, and the 7 larger Diviſions (counted forward) is 700 more, and 3 of the leſſer Diviſions is 30 more, and half of one of the leſſer Diviſions is 5 more, which is the Point repreſenting 2735. You muſt remember, that between each Figure upon the Line there are ten Parts; which I call the larger Diviſions, and each of thoſe larger Diviſions are ſubdivided (or ſuppoſed ſo to be) into ten other Parts, which I call the ſmaller Diviſions, and each of thoſe Parts ſuppoſed to be ſubdivided again into ten other Parts, *&c.* You muſt alſo remember, that if 1, in the middle of the Line, ſtands only for 1, then 1 at the upper End will be 10, and one at the lower End will only be $\frac{1}{10}$; but if 1 at the lower End ſignifies 1, then 1 in the Middle ſtand for 10, and 1 at the upper End is 100, *&c.*

There is one Thing more which I would have my Reader to underſtand, and that is, How to find all ſuch proportional Numbers made Uſe of in the Proportions about a Circle, and of a Cylinder, and in other Places; which Thing may be of good Uſe, to know how to correct a Number which may happen to be falſe printed, or to enlarge any Number to more decimal Places, for more Exactneſs; for though I have mentioned what ſuch Nnmbers are, yet I have not ſhewn how to find them, which a Learner may be a little at a Nonplus to do, though they are eaſily found by the Rules there laid down. I ſhall therefore give two or three Examples, in this Place, of finding ſuch Numbers, which may enable my Reader to find out the reſt.

And, firſt, let it be required to find the Area of a Circle, whoſe Diameter is an Unit.

By the Proportion of *Van Culen*, if the Diameter be 1, the Circumference will be 3.1415926, &c, whereof 3.1416 is ſufficient in moſt Caſes. Then the Rule teaches to multiply half the Circumference by half the Diameter, and the Product is the Area, that is, multiply 1.570 by .5, (*viz.* Half 3.1416 by Half 1) and the Product .7854, which is the Area of the Circle, whoſe Diameter is 1;

Again, if the Area be required, when the Circumference is 1; firſt, find what the Diameter will be, thus, As 3.1416 : to 1 : : ſo is 1 : to .318309, which is the Diameter when the Circumference is 1. Then multiply half .318309 by half 1, that is, .159154 by .5, and the Product is .079577, which is the Area of a Circle whoſe Circumference is 1.

If the Area be given to find the Side of the Square equal, you need but extract the

Square Root of the Area given, and it is done: So the Square Root of .7854 is .8862, which is the Side of a Square equal when the Diameter is 1. And if you extract the Square Root of .079577, it will be .2821, which is the Side of the Square equal to the Circle whoſe Circumference is 1.

If the Side of a Square within a Circle be required, if you Square the Semidiameter and double that Square, and out of that Sum extract the Square Root, that ſhall be the Side of the Square which may be inſcribed in that Circle: ſo, if the Diameter of the Circle be 1, then the Half is .5; which ſquared is .25, and this doubled is .5, whoſe Square Root is .7071, the Side of the Square inſcribed.

Again, if the Diameter of a Globe be 1, to find the Solidity. In Sect XI. Chap. II. it is demonſtrated, that the Globe is $\frac{2}{3}$ of a Cylinder of the ſame Diameter and Altitude: Thus, if the Cylinder's Diameter be 1, and its Altitude or Length be alſo 1, find the Solidity thereof, and take $\frac{2}{3}$ of it, and that will be the Solidity of the Globe required. Now, if the Diameter be 1, the Area of the Circle or Baſe of the Cylinder, is .7854 (as is above ſhewn) which multiplied by 1, the Altitude of the Cylinder, and the Product is alſo .7854; the Solidity of the Cylinder,

$\frac{2}{3}$ whereof is .5236, which is the Solidity of the Globe whoſe Diameter is 1.

From what has been ſaid, the Reader may eaſily perceive how all other proportional Numbers are found, and may examine them at his Pleaſure.

I ſhall not enlarge any farther upon the Matter, but leave the Book to ſpeak for itſelf; and if it prove beneficial to the ingenious Practitioners, I have my Deſire. So, wiſhing my ingenious Reader good Succeſs in his Endeavours, not doubting but he will reap Profit hereby; which that he may, is the hearty Deſire of his Well-wiſher,

W. HAWNEY.

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Complete Meaſurer.

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Jas. Slack Nicholson 24th June 1842

PART I.

CHAP. I.

Notation of DECIMALS.

A Decimal Fraction is an artificial Way of ſetting down and expreſſing Natural or Vulgar Fractions, as whole Numbers: And whereas the Denominators of Vulgar Fractions are diverſe, the Denominators of Decimal Fractions are always certain: For a Decimal Fraction hath always for its Denominator an Unit, with a Cypher or Cyphers annexed to it, and muſt therefore be either 10, 100, 1000, 10000, *&c.* And therefore, in writing down a Decimal Fraction there is no Neceſſity of writing down the Denominator; for by bare Inſpection it is certainly known; it conſiſting of an Unit, with as many Cyphers annexed to it as there are Places (or Figures) in the Numerator.

Example. This Decimal Fraction $\frac{25}{100}$ may be written thus, .25, its Denominator being known to be an Unit with two Cyphers; becauſe there are two Figures in the Numerator. In like manner, $\frac{125}{1000}$ may be thus written, .125, $\frac{75}{1000}$ thus, .3575; and $\frac{3575}{10000}$ thus, .075, and $\frac{65}{10000}$ thus, .0065.

As whole Numbers increaſe in a Decuple, or tenfold Proportion, towards the Left Hand, ſo, on the contrary, Decimals decreaſe, towards the Right Hand, in a decuple Proportion, as in the following Scheme.

Tens of Millions.	Millions.	Hundreds of Thouſands.	Tens of Thouſands.	Thouſands.	Hundreds.	Tens.	Units.	Tenth Parts.	Hundredth Parts.	Thouſandth Parts.	Ten Thouſandth Parts.	Hundred Thouſandth Parts.	Millionth Parts, &c.
7	6	5	4	3	2	1	0	.1	2	3	4	5	6

Hence it appears, that Cyphers put on the Right Hand of whole Numbers, do increaſe the Value of thoſe Numbers in a decuple (or tenfold) Proportion; but being annexed to the Right Hand of a Decimal Fraction, do neither increaſe nor decreaſe the Value thereof: So $\frac{2500}{10000}$ is equivalent to $\frac{25}{100}$ or .25. And, on the contrary, tho' in whole Numbers, Cyphers prefixed before them do neither increaſe nor diminiſh the Value; yet Cyphers before a Decimal Fraction, do diminiſh its Value in a decuple Proportion: For .25 if you prefix a Cypher before it, becomes $\frac{025}{1000}$, or .025; and .125 is $\frac{00125}{100000}$ by prefixing two Cyphers before it, thus, .0025. And therefore, when you are to write a Decimal Fraction, whoſe Denominator hath more Cyphers than there are Figures in the Numerator, they muſt be ſupplied by prefixing ſo many Cyphers before the Figures of your Numerator; as ſuppoſe $\frac{19}{1000}$ were to be written down without its Denominator; here, becauſe there are three Cyphers in the Denominator, and but two Figures in the Numerator, therefore prefix a Cypher before 19, and ſet it down thus, .019.

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The Integers are ſeparated from the Decimals ſeveral Ways, according to Mens Fancies; but the beſt and moſt uſual Way is by a Point or Period; and if there be no whole Number, then a Point before the Fraction is ſufficient: Thus, if you were to write down $317\frac{217}{1000}$ It may be thus expreſſed, 317.217; and $59\frac{25}{10000}$ thus, 59.0025; and $\frac{75}{10000}$ thus, .0075, *&c.*

CHAP. II.

Reduction of DECIMALS.

IN *Reduction of Decimals*, there are three Caſes; *1ſt*, To reduce a Vulgar Fraction to a Decimal. *2dly*, To find the Value of a Decimal in the known Parts of Coin, Weights, Meaſures, *&c.* *3dly*, To reduce Coin, Weights, Meaſures, *&c.* to a Decimal. Of theſe in their Order.

I. *To reduce a Vulgar Fraction to a Decimal.*

The RULE.

As the Denominator of the given Fraction is to its Numerator, ſo is an Unit [with a competent Number of Cyphers annexed] to the Decimal required.

Therefore, if to the Numerator given, you annex a competent Number of Cyphers, and divide the Reſult by the Denominator, the Quotient is the Decimal equivalent to the Vulgar Fraction given.

Example 1. Let $\frac{3}{4}$ be given, to be reduced to a Decimal of two Places, or having 100 for its Denominator.

To 3 (the Numerator given) annex two Cyphers, and it makes 300; which divide by the Denominator 4, and the Quotient is .75, the Decimal required, and is equivalent to $\frac{3}{4}$ given.

N O T E, That ſo many Cyphers as you annex to the given Numerator, ſo many Places muſt be pricked off in the Decimal found; and if it ſhall happen that there are not ſo many Places of Figures in the Quotient, the Deficiency muſt be ſupplied, by prefixing ſo many Cyphers before the Quotient Figures, as in the next *Example*.

Example 2. Let $\frac{3}{573}$ be reduced to a Decimal having ſix Places.

To the Numerator annex ſix Cyphers, and divide by the Denominator, and the Quotient is .5235; but it was required to have ſix Places, therefore you muſt prefix two Cyphers before it, and then it will be .005235, which is the Decimal required, and is equivalent to $\frac{3}{573}$.

See the Work of theſe two *Examples*.

```
4)3.00(.75        573)3.000000(5235
   28                     ....
  ----               ------------
    20               1350
    20                2040
   ---                 3210
    ..               ------------
                          345
```

In the ſecond *Example* there remains 345, which Remainder is very inſignificant, it being leſs than $\frac{1}{1000000}$ Part of an Unit, and therefore is rejected.

II. *To*

II. *To find the Value of a Decimal in the known Parts of Money, Weight, Meaſures, &c.*

The RULE.

Multiply the given Decimal by the Number of Parts in the next inferior Denomination, and from the Product prick off ſo many Places to the Right Hand as there were Places in the Decimal given; and multiply thoſe Figures pricked off by the Number of Parts in the next inferior Denomination, and prick off ſo many Places as before, and ſo continue to do, till you have brought it to the loweſt Denomination required.

Example 1. Let .7565 of a Pound Sterling be given to be reduced to Shillings, Pence and Farthings.

Multiply by 20, by 12, and 4, as the Rule directs, and always prick off four Places to the Right Hand, and you will find it to make 15*s.* 1*d.* 2*q.*

See the Work.

```
     .7565
        20
s.  ------
   15.1300
        12
d.  ------
    1.5600
         4
q.  ------
    2.2400
```

A more compendious Way of finding the Value of the Decimal of a Pound Sterling.

Double the firſt Figure, (or Place of Primes) and it makes ſo many Shillings; and if the next Figure (or Place of Seconds be 5, or more than 5, for the 5 add another Shilling to the former Shillings; then for every Unit in the ſecond Place count ten, and to that add the Figure in the third Place, and reckon them ſo many Farthings; but if they make above 13, abate 1, and if it be above 38, abate 2, and add the remaining Farthings to the Shillings before found.

Example 1. Let .695 of a Pound be reduced to Shillings, Pence, and Farthings.

Firſt, Double your 6, and it makes 12*s.* then take 5 out of 9, and for that reckon another Shilling, and it makes 13*s.* and the four remaining is 4 Tens, and the 5 makes 45, which being above 38, you muſt therefore caſt away 2, and there reſt 43 Farthings, which is 10*d.* $\frac{3}{4}$. So the Anſwer is 13*s.* 10*d.* $\frac{3}{4}$.

	l.	*s.*	*d.*
So the Value of .725 =		14	6
And the Value of .878 =		17	6$\frac{3}{4}$
And the Value of .417 =		8	4

And ſo of any other.

Let .59755 of a Pound Troy be reduced to Ounces, Penny-weights, and Grains.

Multiply by 12, by 20, and by 24, and always prick off five Places towards the right Hand, and you will find the Anſwer to be 7*oz.* 3*pwt.* 10*gr ferè.*

See the WORK.

```
 .59755
     12
 ------
7.17060
     20
 ------
3.41200
     24
 ------
 164800
 82400
 ------
9.88800
```

	oz.	*pwt.*	*gr.*
Facit	7	3	9.888.

Let .43569 of a Tun be reduced to Hundreds, Quarters, and Pounds.

Multiply by 20, by 4, and by 28, and the Anſwer will be 8*C.* 2*qrs.* 14*lb.* *ferè*,

.43569 20	
8.71380 4	*C.* *qrs.* *lb.* *Facit* 8 2 23.9456.
2.85520 28	
23.94560	

Let .9595 of a Foot be reduced into Inches and Quarters.

.9595 12	
11.5140 4	*Facit* 11 Inches, 2 Quarters.
2.0560	

III. *To reduce the known Parts of Money, Weight, Meaſure, &c. to a Decimal.*

The RULE.

To the Number of Parts of the leſſer Denomination given, annex a competent Number of Cyphers, and divide by the Number of ſuch Parts that are contained in the greater Denomination, to which the Decimal is to be brought; and the Quotient is the Decimal ſought.

Example 1. Let 6*d.* be reduced to the Decimal of a Pound.

To 6 annex a competent Number of Cyphers (ſuppoſe 3) and divide the Reſult by 240 (the Pence in a Pound) and the Quotient is the Decimal required.

240)6.000(.25

1200

Facit .025

Example 2. — *Example* 2. Let 3*d.* $\frac{3}{4}$ be reduced to the Decimal of a Pound, having ſix Places.

In 3*d.* $\frac{3}{4}$ there are 15 Farthings; therefore to 15 annex ſix Cyphers (becauſe there are to be ſix Places in the Decimal required) and divide by 960 (the Farthings in a Pound) and the Quotient is .015625.

96|0)15.00000|0(.015625.

540

600

240

480

Example 3. Let 3 $\frac{1}{4}$ Inches be reduced to the Decimal of a Foot, conſiſting of four Places.

In 3 $\frac{1}{4}$ Inches there are 13 Quarters; therefore to 13 annex four Cyphers, and divide by 48, (the Quarters in a Foot) and the Quotient is .2708.

48)13.0000(.2708

340

400

16

Example 4. Let 9*C.* 1*qr.* 16*lb.* be reduced to the Decimal of a Tun, having ſix Places.

```
C.  qr.  lb.
 9   1   16          224'0)1052.00000'0(.469642
 4                             .....
———                       ———————————
 37 qrs.                   15600
 28                         21600
———                          14400
302   Facit .469642.           9600
 75                             6400
———                         ————————
1052 Pounds.                    1920
```

C H A P. III.

Addition of DECIMALS.

ADDITION of Decimals is performed the ſame Way as Addition of whole Numbers, only you muſt obſerve to place your Numbers right, that is, Units under Units, Primes under Primes, Seconds under Seconds, *&c.*

Example. Let 317.25, 17.125, 275.5, 47.3579, and 12.75 be added together into one Sum.

```
      317.25
       17.125
      275.5
       47.3579
       12.75
     ————————
Sum   669.9829
```

This is ſo plain, that more Examples I think needleſs.

CHAP. IV.

Subtraction of DECIMALS.

SUBTRACTION of Decimals is performed likewiſe the ſame Way as in whole Numbers, Reſpect being had to the right placing the Numbers, [as in Addition] as in the following *Examples.*

	(1)		(2)
From	212.0137	From	201.1250
Subtr.	31.1275	Subtr.	5.5785
Reſts	180.8862	Reſts	195.5465
Proof	212.0137	Proof	201.1250

	(3)		(4)
From	2051.315	From	30.5
Subtr.	79.172	Subtr.	7.2597
Reſts	1972.143	Reſts	23.2403
Proof	2051.315	Proof	30.5

NOTE, If the Number of Places in the Decimals be more in that which is to be ſubtracted, than in that which you ſubtract from, you muſt ſuppoſe Cyphers to make up the Number of Places; as in the fourth *Example.*

CHAP. V.

Multiplication of DECIMALS.

MULTIPLICATION of Decimals is alſo performed the ſame Way as Multiplication of whole Numbers; but to know the Value of the Product, obſerve this

RULE.

" Cut off, or ſeparate by a Comma, or Prick, ſo
" many Decimal Places in the Product, as there are
" Places of Decimals in both Factors, *viz.* in the
" Multiplicand and Multiplier, which I ſhall farther
" explain in the following.

Examples.

Let 3.125 be multiplied by 2.75; multiply the Numbers together, as if they were whole Numbers, and the Product is 8.59375: And becauſe there were three Places of Decimals pricked off in the Multiplicand, and two Places in the Multiplier, therefore you muſt prick off five Places of Decimals in the Product, as you may ſee by the Work.

```
  3.125
   2.75
 ------
  15625
 21875
 6250
 ------
8.59375
```

Let 79.25 be multiplied by .459.

In this *Example*, becauſe two Places of Decimals are prick'd off in the Multiplicand, and three in the Multiplier, therefore there muſt be five prick'd off in the Product.

```
   79.25
    .459
  ------
   71325
  39625
 31700
 -------
 36.37575
```

Let .135272 be multiplied by .00425.

In this *Example*, becauſe in the Multiplicand are ſix Decimal Places, and in the Multiplier five Places, therefore in the Product there muſt be eleven Places of Decimals: but when the Multiplication is finiſhed, the Product is but 57490600, *viz*, only eight Places; therefore, in this Caſe, you muſt prefix three Cyphers before the Product Figures to make up the Number of eleven Places; ſo the true Product will be ,00057490600.

```
      .135272
       .00425
     --------
       676360
      270544
     541088
  -----------
 .00057490600
```

More Examples for Practice.

.001472
.1045

7360
5888
1472

.0001538240

279.25
.445

139625
111700
111700

124.26625

4.443
15.98

35544
39987
22215
4443

70.99914

7.3564
.0126

441384
147128
73564

.09269064

.017532
347

122724
70128
52596

6.083604

32.0752
0.325

1603760
641504
962256

1.04244400

20.0292
35.45

1001455
801164
1001455
600873

710.031595

.75432
.0356

452592
377160
226296

.026853792

Con-

Contracted Multiplication of Decimals.

Becauſe in Multiplication of Decimal Parts and mixed Numbers, there is no need to expreſs all the Figures of the Product, but in moſt Caſes, two, three, or four Places of Decimals will be ſufficient; therefore, to contract the Work, obſerve this following

RULE.

Write the Units Place of the Multiplier under that Place of the Multiplicand, whoſe Place you intend to keep in the Product; then invert the Order of all the other Figures, that is, write them all the contrary Way: Then, in multiplying, always begin at that Figure in the Multiplicand which ſtands over the Figure you are then multiplying withal, and ſet down the firſt Figure of each particular Product directly one under the other; but yet a due Regard muſt be had to the Increaſe ariſing from the Figures on the Right Hand of that Figure in the Multiplicand which you begin to multiply at. This will appear more plain by *Examples.*

Example 1. Let 2.38645 be multiplied by 8.2175, and let there be only four Places retained in the Decimals of the Product.

Firſt, according to the Directions, write down the Multiplicand, and under it write the Multiplier thus; place the 8 (being the Units Place of the Multiplier) under 4, the fourth Place of Decimals in the Multiplicand, and write the reſt of the Figures quite contrary to the uſual Way, as in the following Work: Then begin to multiply, firſt the five which is left out, (only with Regard to the Increaſe which muſt be carried from it) ſaying, 8 times 5 is 40, carry 4 in your Mind, and ſay 8 times 4 is 32, and 4 I carry, is 36; ſet down 6 and carry 3; and proceed thro' the reſt of the Figures, as in common Multiplication: Then begin to multiply with 2, ſaying, 2 times 4 is 8, for which I carry 1, (becauſe it is above 5) and ſay, 2 times 6 is 12, and 1 that I carry is 13; ſet down 3 and carry 1, and proceed thro' the reſt of the Figures: Then multiply with 1, ſaying, once 6 is 6, for which carry 1, and ſay, once 8 is 8, and 1 is 9; ſet down 9, and proceed: Then multiply with 7, ſaying 7 times 8 is 56, for which carry 6, (becauſe it is above 55) and ſay, 7 times 3 is 21, and 6 that I carry, is 27; ſet down 7 and carry 2, and proceed: Then multiply with 5, ſaying 5 times 3 is 15, for which carry 2, and ſay, 5 times 2 is 10, and 2 I carry, is 12, which ſet down, and add all the Products together, and the total Product will be 19.6107. See the Work.

```
  2.38645
  5712.8
 ---------
 190916
   4773
    239
    167
     12
 ---------
 19.6107
```

NOTE, That in multiplying the Figure left out every Time next the Right Hand in the Multiplicand if the Product be 5, or upwards to 10, you carry 1; and if it be 15, or upwards to 20, carry 2; and if 25, or upwards to 30, carry 3, *&c.*

I have here ſet down the Work of the laſt *Example*, wrought by the common Way, by which you may ſee both the Reaſon and Excellency of this Way, all the Figures on the Right Hand of the Line being wholly omitted.

```
     2.38645
      8.2175
 ---------------
      11|93225
     167|0515
     238|645
    4772|90
  190916|0
 ---------------
 19.6106|52875
```

Example 2. Let 375.13758 be multiplied by 16.7324, ſo that the Product may have but four Places of Decimals.

Firſt ſet 6, the Unit's Place of the Multiplier, under 5, being the fourth Place of Decimals in the Multiplicand, (becauſe four Places of Decimals were to be prick'd off) and write all the reſt of the Figures backward; then multiply all the Figures of the Multiplicand by 1, after the common Way; then begin with the ſecond Figure of the Multiplier 6, ſaying, 6 times [illegible] which I carry 5; (in reſpect of the 8 le[illegible] 6 times 5 is 30, and 5 that I carry is 35; ſet down 5 and carry 3, and proceed after the common Method: Then begin with 7, the third Figure of the Multiplier, and ſay, 7 times 5 is 35, for which carry 4, and ſay, 7 times 7 is 49, and 4 I carry is 53; ſet down 3 under the firſt, and carry 5, and proceed as before: Then begin with 3, the fourth Figure of the Multiplier, and ſay, 3 times 7 is 21, carry 2, and ſay, 3 times 3 is 9, and 2 I carry is 11; ſet down 1 and carry 1, and proceed as before: Then begin with 2, the fifth Figure, and ſay, 2 times 3 is 6, for which I carry 1, and ſay, 2 times 1 is 2, and I carry is 3; ſet down 3, and 2 times 5 is 10; ſet down 0 and carry 1, and proceed as before: Then begin with 4, the laſt Figure of the Multiplier, and ſay, 4 times 1 is 4, for which I carry nothing, becauſe it is leſs than 5; then ſay, 4 times 5 is 20; ſet down 0, and carry 2, and proceed thro' the reſt of the Figures of the Multiplicand: Then add all up together, and the Product is 6276.9520.

See the Work.

375.13758 the Multiplicand.
4237.61 the Multiplier reverſed.

37513750 the Product with 1.
22508255 the Product with 6 increaſed with 6 × 8.
2625963 the Product with 7 increaſed with 7 × 5.
112541 the Product with 3 increaſed with 3 × 7.
7503 the Product with 2 increaſed with 2 × 3.
1500 the Product with 4 increaſed with 0.

6276.9520 the Product required.

Let the ſame *Example* be repeated, and let only one Place in Decimals be pricked off.

375.13758 the Multiplicand.
4237.61 the Multiplier inverted.

37514 the Product by 1 with the Increaſe of 1 × 7.
22508 the Product with 6 increaſed with 6 × 3.
2626 the Product with 7 increaſed with 7 × 1.
113 the Product with 3 increaſed with 3 × 5.
7 the Product with 2 increaſed with 2 × 7.
1 the Increaſe only of 4 × 3.

6276.9 the Product is the ſame as before.

More Examples for Practice.

Multiply 395.3756 by .75642, and prick off four Places in Decimals.

395.3756 the Multiplicand.
24657. the Multiplier reverſed.

2767629 the Product by 7 increaſed with 7 × 6.
197688 the Product by 5 increaſed with 5 × 5.
23722 the Product by 6 increaſed with 6 × 7.
1581 the Product by 4 increaſed with 4 × 3.
79 the Product by 2 increaſed with 2 × 5.

299.0699 the Product required.

Let the ſame *Example* be repeated, and let there be only one Place of Decimals.

395·3756
24657·
2767 the Product by 7 increaſed with 7 × 3.
198 the Product by 5 increaſed with 5 × 5.
24 the Product by 6 increaſed with 6 × 9 + 6 × 5.
2 the Increaſe of 4 × 9 + 4 × 3.

299.1 the Product.

Characters and their Signification.

NOTE, That this Mark + ſignifies Addition; 8 + 5 that is 8 more 5, or 8 added to 5; and 8 + 3 + 7 denotes theſe Numbers are to be added into one Sum.

This Mark — ſignifies Subtraction; as 9—4 ſignifies that 4 is to be taken from 9.

This Mark × ſignifies Multiplication; as 7 × 5 ſignifies that 7 is to be multiplied into 5.

This Mark ÷ ſignifies Diviſion; as 12 ÷ 4 ſignifies 12 is to be divided by 4.

This Mark = ſignifies Equality, or Equation, that is, when = is placed between Numbers or Quantities, it denotes them to be equal; as 7 + 5 = 12, that is, 7 more 5 is equal to 12; and 15—7 = 8, that is, 15 leſs by 7, is equal to 8, or ſubtract 7 from 15 and there remains 8.

This Mark :: is the Sign of Proportion, or the Golden Rule, it being always placed betwixt the two middle Terms or Numbers in Proportion, thus, 4 : 20 :: 6 : 30; to be thus read, as 4 is to 20, ſo is 6 to 30.

CHAP. VI.

Diviſion of DECIMALS.

DIVISION of Decimals is performed after the ſame Manner as Diviſion of whole Numbers; but to know the Value or Denomination of the Quotient is the only Difficulty; for the reſolving of which obſerve either of the following

RULES.

I. The firſt Figure in the Quotient muſt be of the ſame Denomination with that Figure in the Dividend which ſtands (or is to be ſuppoſed to ſtand) over the Unit's Place in the Diviſor, at the firſt ſeeking.

II. When the Work of Diviſion is ended, count how many Places of Decimal Parts there are in the Dividend more than in the Diviſor, for that Exceſs is the Number of Places which muſt be ſeparated in the Quotient for Decimals: But if there be not ſo many Figures in the Quotient, as is the ſaid Exceſs, that Deficiency muſt be ſupplied with Cyphers in the Quotient, prefixed before the ſignificant Figures thereof, towards the Left Hand, with a Point before them; ſo ſhall you plainly diſcover the Value of the Quotient.

Theſe following Directions ought alſo to be carefully obſerved.

If the Diviſor conſiſts of more Places than the Dividend, there muſt be a competent Number of Cyphers annexed to the Dividend, to make it conſiſt of as many (at leaſt) or more Places of Decimals than the Diviſor; for the Cyphers added muſt be reckoned as Decimals.

Conſider whether there be as many decimal Parts in the Dividend as there are in the Diviſor; if there be not, make them ſo many, or more, by annexing of Cyphers.

In dividing of whole or mixed Numbers, if there be a Remainder, you may bring down more Cyphers, and, by continuing your Diviſion, carry the Quotient to as many Places of Decimals as you pleaſe.

These Things being conſidered, I ſhall proceed to the Practice of Diviſion of Decimals, which I ſhall endeavour to explain in as familiar and eaſy a Method as poſſible.

Example 1. Let 48 be divided by 144.

In this *Example* the Diviſor 144 is greater than the Dividend 48; therefore, according to the Directions above, I annex a competent Number of Cyphers, (*viz.* four) with a Point between them, and divide according to the uſual Way,

144) 48.0000 (.3333

480

480

480

48

But firſt, in ſeeking how often 144 in 480, (the firſt three Figures of the Dividend) I find the Unit's Place of the Diviſor to fall under the firſt Place of Decimals; therefore the firſt Figure in the Quotient is in the firſt Place of Decimals: Or, by the ſecond Rule, there being four Places of Decimals in the Dividend, and none in the Diviſor, ſo the Exceſs of decimal Places in the Dividend, above that in the Diviſor, is four; ſo that when the Diviſion is ended, there muſt be four Places of Decimals in the Quotient.

See the Work.

Example 2. Let 217.75 be divided by 65.

Firſt, in ſeeking how often 65 in 217, (the firſt three Figures of the Dividend) I find the Unit's Place of the Diviſor to fall under the Unit's Place of the Dividend; therefore the firſt Figure in the Quotient will be Units, and all the reſt Decimals. Or, by the ſecond Rule, there being two Places of Decimals in the Dividend, and no Decimals in the

Diviſor, therefore the Exceſs of Decimal Places in the Dividend, above the Diviſor is two; ſo when the Diviſion is ended, ſeparate two Places in the Quotient, towards the Right Hand by a Point.

See the Work.

65)217.75(3.35
· · · ·

227
325

· · ·

Example 3. Let 267.15975 be divided by 13.25.

13.25)267.15975(20.163
· · · · · ·

2159
8347
3975

· · · ·

In this *Example*, 3, the Unit's Place of the Diviſor, falls under 6, the Ten's Place of the Dividend; therefore (by the firſt Rule) the firſt Figure in the Quotient is Tens: Or, by the ſecond Rule, the Exceſs of Decimal Places in the Dividend, above the Diviſor, is three; there being five Places of Decimals in the Dividend, and but two in the Diviſor, ſo there muſt be three Places of Decimals in the Quotient.

Example 4. 15.675159 be divided by 375.89.

375.89)15.675159(0417
· ·

63955
263669

546

In this *Example*, 5, the Unit's Place of the Diviſor, falls under 7, the ſecond Place of Decimals in the Dividend; therefore (by the firſt Rule) the firſt Figure in the Quotient is in the ſecond Place of Decimals, ſo that you muſt put a Cypher before the firſt Figure in the Quotient, and by the ſecond Rule, the Exceſs of Decimal Places in the Dividend, above the Number of Decimal Places in the Diviſor, is 4; for the Decimal Places in the Dividend is 6, and the Number of Places in the Diviſor but two; therefore there muſt be four Places of Decimals in the Quotient. But the Diviſion being finiſhed after the common Way, the Figures in the Quotient are but three; therefore you muſt prefix a Cypher before the ſignificant Figures.

Example 5. Let 72.1564 be divided by .1347.

```
.1347)72.1564(535.68
          . . .
      ---------
        4806
         7654
          9190
          11080
          ------
            304
```

In this *Example*, the Diviſor being a Decimal, the firſt Figure thereof falls under the Ten's Place in the Dividend; therefore the Units (if there had been any) ſhould fall under the Hundred's Place in the Dividend, and ſo the firſt Figure in the Quotient is Hundreds: And, by the ſecond Rule, there being four Places of Decimals in the Dividend, and as many in the Diviſor, ſo the Exceſs is nothing; but in dividing, I put two Cyphers to the Remainders, and continue the Diviſion to two Places farther; ſo I have two Places of Decimals.

See the Work.

Exampl

Example 6. Let .125 be divided by .0457.

.0457) .1250000 (2.735
...
914
3360
3199
1610
1371
2390
2285
105

In this *Example*, the Unit's Place of the Diviſor (if there had been any) would fall under the Unit's Place of the Dividend; therefore the firſt Figure of the Quotient is Units: And, by the ſecond Rule, there being ſeven Places of Decimals in the Dividend, and but four Places in the Diviſor, ſo the Exceſs is three; therefore there muſt be three Places of Decimals in the Quotient.

I ſhall ſet down only the Work of ſome few *Examples* more, and ſo proceed to *Contracted Diviſion.*

.00456) .0000059791 (.00131
..
1419
511
55

Let 1 be divided by 282, *&c.*

```
282) 1.0000000 (.0035461 ferè.
          . . . . .
       ___________
        1540
         1300
          1720
           280

325) .400000 (1.2307
          . . .
       ___________
        750
        1000
         2500
       ___________
          225

.042) 495.00000 (11785.71
          . . . . .
       ___________
        75
        330
         360
          240
           300
            60
        ___________
            18
```

Diviſion of DECIMALS *contracted.*

IN Diviſion of Decimals the common Way, when the Diviſor hath many Figures, and it is required to continue the Diviſion till the Value of the Remainder be but ſmall, the Operation will ſometimes be large and tedious, but may be excellently contracted by the following Method.

The RULE.

By the firſt Rule of this Chapter, *(pag.* 19.) find what is the Value of the firſt Figure in the Quotient; then, by knowing the firſt Figure's Denomination, you may have as many or as few Places of Decimals as you pleaſe, by taking as many of the Left Hand Figures of the Diviſor as you think convenient for the firſt Diviſor; and then take as many Figures of the Dividend as will anſwer them; and in dividing, omit one Figure of the Diviſor at each following Operation.

A few *Examples* will make it plain.

Example 1. Let 721.17562 be divided by 2.25743, and let there be three Places of Decimals in the Quotient.

```
2.25743)721.175|62(319.467
   · · · · · 677229
             ------
              43946
              22574
              -----
              21372
              20317
              -----
               1055
                903
               ----
                152
                135
                ---
                 17
                 15
                 --
                  2
```

In this *Example*, the Unit's Place of the Diviſor falls under the Hundred's Place in the Dividend; and it is required that three Places of Decimals be in the Quotient; ſo there muſt be ſix Places in all, that is, three Places of whole Numbers, and three Places of Decimals: Then, becauſe I can have the Diviſor in the firſt ſix Figures of the Dividend, I cut off the 62 with a Daſh of the Pen, as uſeleſs; then I ſeek how oft the Diviſor in the Dividend, and the Anſwer is three times; put 3 in the Quotient, and multiply and ſubtract as in common Diviſion, and the Remainder is 43946: Then prick off the 3 in the Diviſor, and ſeek how oft the remaining Figures may be had in 43946, the Remainder, which can be but once; put 1 in the Quotient, and multiply and ſubtract, and the next remainder is 21372: Then prick off the 4 in the Diviſor, and ſeek how often the remaining Figures may be had in 2372, which will be nine times; put 9 in the Quotient; multiply thus, ſaying, 9 times 4 is 36, for which I carry 4, (in reſpect of the 4 laſt pricked off) and 9×7 is 63, and 4 is 67; ſet down 7 and carry 6, and ſo proceed till the Diviſion be finiſhed, always reſpecting the Increaſe made from the Figures pricked off. Obſerve the Work, which will better inform you than many Words.

2.25743)721.17562(319.467

677229	
43946	6
22574	3
21372	32
20316	87
1055	450
902	972
152	4780
135	4458
17	03220
15	80201
1	23019

I have ſet down the Work of this laſt *Example* at large, according to the common Way, that thereby the Learner may ſee the Reaſon of the Rule, all the Figures on the Right Side of the Perpendicular Line being wholly omitted.

Example 2. Let 5171.59165 be divided by 8.758615, and let it be required that four Places of Decimals be pricked off in the Quotient.

```
8.758615(5171.5916|5(590.4577
••••••          •
           43793075
           --------
            7922841
            7882754
            -------
              40087
              35034
              -----
               5053
               4379
               ----
                674
                613
                ---
                 61
                 61
                 --
                 ••
```

In this *Example*, I can't have 8, the firſt Figure in the Diviſor, in 5, the firſt Figure of the Dividend; ſo that the Unit's Place of the Diviſor falls under the Hundred's Place of the Dividend; ſo that there will be ſeven Figures in the Quotient, that is three of whole Numbers, and four of Decimals; therefore there muſt be 7 Figures in the Diviſor, (becauſe the Number of Places in the Diviſor and Quotient will be equal) and there muſt be eight Places in the Dividend, ſo that I cut off the Figure 5 with a Daſh, as uſeleſs. Thus having proportioned the Dividend to the Diviſor, and both to the Number of Places or Figures deſired in the Quotient, I proceed to divide as before, ſaying, how often 8 in 51, which will be 5 times; put 5 in the Quotient, and multiply and ſubtract, and the Remainder is 7922841: Then I prick off the firſt Figure in the Diviſor, 5, and ſeek how often the remaining Figures of the Diviſor in the aforeſaid remainder, which I find 9 times; put 9 in the Quotient, and multiply thereby, ſaying, 9 times 5 (the Figure pricked off is 45, for which I carry 5, and ſay, 9 times 1 is 9, and 5 I carry is 14; ſet down 4, and carry 1, and proceed to multiply the reſt of the Figures, and ſubtract, and the Remainder will be 40087: Then prick off the Figure 1, and ſeek how often 87586 in the Remainder 40087, the Anſwer will be 0; ſo put 0 in the Quotient, and prick off the Figure 6, and ſeek how often 8758 in 40087, which will be 4 times; put 4 in the Quotient, and multiply, ſaying, 4 times 6 (the Figure laſt prick'd off) is 24, for which I carry 2, and ſay, 4 times 8 is 32, and 2 I carry is 34; ſet down 4 and carry 3; multiply the reſt of the Figures, and ſubtract as before; and ſo proceed after the ſame Manner; until all the Figures of the Diviſor be prick'd off to the laſt Figure.

See the Work.

Example 3. Let 25.1367 be divided by 217.3543, and let there be five Places of Decimals in the Quotient.

In this *Example*, 7, the Unit's Place of the Diviſor, falls under 1, the firſt Place of Decimals, therefore the firſt Figure of the Quotient is in the firſt Place of Decimals, ſo the Quotient will be all Decimals: Then, becauſe the Quotient Figures and the Figures of the Diviſor will be of an equal Number, daſh off the 43 in the Diviſor, and the 7 in the Dividend, as uſeleſs, and divide as before.

```
217.35|43)25.136|7).11564
          21735
          -----
           3401
           2174
           ----
           1227
           1087
           ----
            140
            130
            ---
             10
              8
             --
              2
```

Altho' I have hitherto given Directions for proportioning the Diviſor and Dividend, ſo as to bring into the Quotient what Number of Decimals you pleaſe, yet there is no abſolute Neceſſity for it; but you may carry on your Diviſion to what Degree you pleaſe, before you begin to prick off the Figures of the Diviſor, in order to contract the Work, as in the following *Examples*, where it is not required to prick off any determinate Number of Decimals, but it may be done according to Diſcretion.

```
2.756756)7414.76717(2689.67118
 ......       ...
         5513512
         ---------
         19012551
         16540536
         ---------
         24720157
         22054048
         ---------
          2666109
          2481080
          --------
           185029
           165405
           -------
            19624
            19297
            ------
              327
              276
             -----
               51
               28
             -----
               23
               22
             -----
                1
```

12.34254

```
12.34254)514.75498(41.705757
      ···  4937016
           -------
           2105338
           1234254
           -------
            871084
            863978
           -------
              7106
              6171
              ----
               935
               864
               ---
                71
                62
                --
                 9
                 8
                 -
                 1
```

CHAP. VII.

Extraction of the SQUARE-ROOT.

If a Square *Number be given,*

TO find the Root thereof; that is, to find out ſuch a Number, as being multiplied into itſelf, the Product ſhall be equal to the Number given, ſuch Operation is called, *The Extraction of the* SQUARE ROOT; which to do, obſerve the following Directions.

1ſt, You muſt point your given Number, that is, make a Point or Prick over the Unit's Place, another upon the hundred's, and ſo upon every ſecond Figure throughout.

2dly, Then ſeek the greateſt ſquare Number in the firſt Point towards the Left Hand, placing the Square Number under the firſt Point, and the Root thereof in the Quotient, and ſubtract the ſaid Square Number from the firſt Point, and to the Remainder bring down the next Point, and call that the Reſolvend.

3dly, Then double the Quotient, and place it for a Diviſor, on the Left Hand of the Reſolvend, and ſeek how often the Diviſor is contained in the Reſolvend, (reſerving always the Unit's Place,) and put the Anſwer in the Quotient, and alſo on the Right Hand Side of the Diviſor; then multiply by the Figure laſt put in the Quotient, and ſubtract the Product from the Reſolvend (as in common Diviſion) and bring down the next Point to the Remainder (if there be any more) and proceed as before.

A TABLE *of Squares and Cubes, and their Roots.*

Root	1	2	3	4	5	6	7	8	9
Squa.	1	4	9	16	25	36	49	64	81
Cube	1	8	27	64	125	216	343	512	729

Example 1. Let 4489 be a Number given, and let the Square Root thereof be required.

$$\dot{4}4\dot{8}9(67$$

36

127)889 Reſolvend
    889 Product

...

Firſt, point the given Number, as before directed; then (by the little Table aforegoing) ſeek the greateſt ſquare Number in 44, (the firſt Point to the Left Hand) which you will find to be 36, and 6 the Root; put 36 under 44, and 6 in the Quotient, and ſubtract 36 from 44, and there remains 8: Then to that 8 bring down the other Point 89, placing it on the Right Hand, ſo it makes 889 for a Reſolvend; then double the Quotient 6, and it makes 12, which place on the Left Hand for a Diviſor, and ſeek how often 12 in 88 (reſerving the Unit's Place) the Anſwer is 7 times, which put in the Quotient; and alſo on the Right Hand Side of the Diviſor, and multiply 127 by 7, (as in common Diviſion) and the Product is 889, which ſubtracted from the Reſolvend, there remains nothing; ſo is your Work finiſhed, and the ſquare Root of 4489 is 67; which Root, if you multiply by itſelf, that is, 67 by 67, the Product will be 4489, equal to the given ſquare Number, and proves the Work to be right.

Example 2. Let 106929 be a Number given, and let the Square Root thereof be required.

106929(327

9

62) 169 Reſolvend.

124 Product.

647) 4529 Reſolvend.

4529 Product.

Firſt, point your given Number, as before directed, putting a Point upon the Units, Hundreds, and Tens of Thouſands; then ſeek what is the greateſt ſquare Number in 10 (the firſt Point) which by the little Table you will find to be 9, and 3 the Root thereof; put 9 under 10, and 3 in the Quotient; then ſubtract 9 out of 10, and there remains 1, to which bring down 69, the next Point, and it makes 169 for the Reſolvend; then double the Quotient 3, and it makes 6, which place on the Left Hand of the Reſolvend for a Diviſor, and ſeek how often 6 in 16, the Anſwer is twice; put 2 in the Quotient;

tient, and alſo on the Right Hand of the Diviſor, making it 62: Then multiply 62 by the 2 you put in the Quotient, and the Product is 124; which ſubtract from the Reſolvend, and there remains 45; to which bring down 29, the next Point, and it makes 4529 for a new Reſolvend: Then double the Quotient 32, and it makes 64, which place on the Left Side the Reſolvend for a Diviſor, and ſeek how oft 64 in 452, which you will find 7 times; put 7 in the Quotient, and alſo on the Right Hand of the Diviſor, making it 647, which multiplied by 7 in the Quotient, it makes 4529; which ſubtracted from the Reſolvend, there remains nothing: So 327 is the ſquare Root of the given Number.

Example 3. Let 2268741 be a ſquare Number given, the Root whereof is required.

```
 ·  ·  ·
2268741 (1506.23
1
──────
25)126
   125
──────────
3006) 18741
      18036
──────────
30122)70500
      60244
──────────
301243)1025600
       .903729
──────────
Remains .121871
```

Having pointed the given Number, as before directed, ſeek what is the greateſt ſquare Number in the firſt Point 2, which is 1; put 1, the Square, under 2, and 1, the Root thereof, in the Quotient; ſubtract 1 from 2, and there remains 1; to which bring down the next Point 26, and ſet on the Right Hand, making it 126; double the 1 in the Quotient, which makes 2; ſet 2 on the Left Hand for a Diviſor, and aſk how often 2 in 12, which will be five times; put 5 in the Quotient, and alſo on the Right Hand of the Diviſor, making it

25; multiply (as in common Diviſion) 25 by 5, and ſubtract the Product, 125, from 126, and there remains 1: Bring down the next Point, 87, and it makes 187 for a new Reſolvend, and double the 15 in the Quotient, it makes 30 for a new Diviſor: Then ſeek how often 30 in 18, which you can't have; ſo that you muſt put 0 in the Quotient, and alſo on the Right Hand of the Diviſor, and bring down the next Point, and it makes 18741 for another new Reſolvend: Then ſeek how often 300 in 1874, which will be 6 times; put 6 in the Quotient, and alſo on the Right Hand of the Diviſor, multiply and ſubtract, and the Remainder will be 705. Now, if you have a mind to find the Value of the Remainder, you may annex Cyphers, by two at a time to the Remainders, and ſo proſecute the Work to what Number of decimal Parts you pleaſe; thus, to 705 annex two Cyphers, and it will make 70500, and the Quotient doubled, is 3012 for a Diviſor; Then ſeek how often 3012 in 7050 (rejecting the Unit's Place) which will be twice; put 2 in the Quotient, and alſo on the Right Hand of the Diviſor, and multiply and ſubtract as before, and the Remainder will be 10256; to which annex two Cyphers, and proceed as before, and you will get 3 in the Quotient next. So the ſquare Root of the given Number is 1506.23, which being ſquared or multiplied by itſelf, and the laſt Remainder added, will make the given Number as follows:

```
                              1506.23
                              1506.23
                              -------
                               451869
                              301246
                             903738
                            753115
                           150623
                          -----------
                          2268728.8129
The Remainder add ----------12.1871
                          -----------
                  Proof   2268741.0000
```

Some more Examples for Practice.

. . . .

Example 1. 7596796 (2756.128 Root.

4

47) 359

329

545) 3067

2725

5506) 34296

33036

55122) 126000

110244

551242) 1575600

1102484

5512448) 47311600

44099584

3212016

More Examples for Practice.

Example 2. 751417.5745(866.84 Root.

```
         751417.5745(866.84 Root.
         64
        ------
   166)1114
        996
       ------
  1726)11817
        10356
       -------
 17328)146157
        138624
       --------
173364)753345
        693456
       --------
         59889
       --------
```

If the given Number be a mixed Number, *viz.* conſiſting of a whole Number and a Decimal together, make the Number of decimal Places even, that is 2, 4, 6, 8, *&c.* that ſo there may a Point fall upon the Unit's Place of the whole Numbers, as in this laſt *Example,* and in that following.

Example 3. Let 656714.37512 be given, to find the Square-Root.

· · · · · · ·
656714.375120 (810.379 Root.
64

161)167
161

16203)61437
48609

162067)1282851
1134469

1620749)14838220
14586741

Remains 251479

In this *Example* there are five Places of Decimals; put a Cypher to it, to make it even, that ſo there may a Point fall upon 4, the Unit's Place.

To find the Square Root of a Fraction.

If it be a Decimal Fraction, the Work differs nothing from the *Examples* foregoing, only you muſt be mindful to point your given Number aright; for (as was before directed) the Number of Places muſt always be made even, and then begin to point at the Right Hand, as in whole Numbers.

If it be a Vulgar Fraction, it muſt be reduced to a Decimal, by the firſt Rule of the ſecond Chapter.

I ſhall give an *Example* or two in each Caſe, and ſo conclude this Chapter.

Let .125 be a Decimal Fraction given, whoſe ſquare Root is required; and let it be required to have four Places of Decimals in the Root.

. . . .
.12500000(.3333
9

65)350
325

703)2500
2109

7065)39100
35325

3775

In this *Example* there muſt be five Cyphers annexed, becauſe two Places in the Square make but one in the Root.

Let the Square Root of .00715 be required.

.007150(.084
64

164)750
656

94

In this a Cypher is added to make the Places even.

Let $\frac{7}{8}$ be a vulgar Fraction given, whoſe Square Root is requir'd.

```
                                     . . . .
 8) 7000                           ).87500000(.9354
    64                               81
    ——                               ——
    60                          183) 650
    56                               549
    ——                               ———
     40                        1865) 10100
     40                               9325
     ——                               ————
     ..                       18704) 77500
                                      74816
                                      —————
                                       2684
```

Reduce this $\frac{7}{8}$ to a Decimal, it makes .875, to which annex Cyphers, and extract the Square Root, as if it was a whole Number. So the Root is .9354.

Let $\frac{3}{960}$ be a Vulgar Fraction, whoſe Square Root is required.

```
                                            . . . . Root.
 96|0) 3.0000000|0                        (.00312500(.0559
           . . . . .                             25
       288                                       ——
       ———                                 105) 625
        120                                      525
         96                                      ———
         ——                               1109) 10000
         240                                     9981
         192                                     ————
         ———                                       19
          480
          480
          ———
```

In extracting the Root of this, becauſe the firſt Point conſiſts of Cyphers, there muſt be a Cypher put firſt in the Quotient.

To prove this Rule, ſquare the Root, and to the Product add the Remainder, as was before directed, To ſquare a Number is to multiply it by itſelf: and to cube it is to multiply the Square of the Number by the Number itſelf.

CHAP. VIII.

Extraction of the CUBE ROOT.

TO extract the CUBE ROOT, is nothing elſe but to find ſuch a Number, as being firſt multiplied into itſelf, and then into that Product, produceth the given Number; which to perform, obſerve theſe following Directions.

1*ſt*, You muſt point your given Number, beginning with the Unit's Place, and make a Point or Dot over every third Figure towards the Left Hand.

2*dly* Seek the greateſt Cube Number in the firſt Point, towards the Left Hand, putting the Root thereof in the Quotient, and the ſaid Cube Number under the firſt Point, and ſubtract it therefrom, and to the Remainder bring down the next Point, and call that the Reſolvend.

3*dly*, Triple the Quotient, and place it under the Reſolvend, the Units Place of this under the Ten's Place of the Reſolvend; and call this the Triple Quotient.

4thly, Square the Quotient, and triple the Square, and place it under the triple Quotient; the Units of this under the Ten's Place of the triple Quotient, and call this the triple Square.

5thly, Add theſe together, in the ſame Order as they ſtand, and the Sum ſhall be the Diviſor.

6thly, Seek how often the Diviſor is contained in the Reſolvend, rejecting the Unit's Place of the Reſolvend (as in the Square Root) and put the Anſwer in the Quotient.

7thly, Cube the Figure laſt put in the Qotient, and put the Unit's Place thereof under the Unit's Place of the Reſolvend.

8thly, Multiply the Square of the Figure laſt put in the Quotient, into the triple Quotient, and place the Product under the laſt, one Place more to the Left Hand.

9thly, Multiply the triple Square by the Figure laſt put in the Quotient, and place it under the laſt, one Place more to the Left Hand.

10thly, Add the three laſt Numbers together, in the ſame Order as they ſtand, and call that the Subtrahend.

Laſtly, Subtract the Subtrahend from the Reſolvend, and if there be another Point, bring it down in the Remainder, and call that a new Reſolvend, and proceed in all reſpects as before, from the Beginning of the third Step to the End of this laſt.

Example 1. Let 314432 be a cubick Number, whoſe Root is required.

$31\dot{4}43\dot{2}$(68 Root
216

98432 Reſolvend.

18 Triple Quotient of 6.

108 Triple Square of the Quotient 6.

1098 Diviſor.

512 Cube of 8, the laſt Figure of the Root.
1152 The Square of 8, by the triple Quotient.
854 The triple Square of the Quotient 6 by 8.

98432 The Subtrahend.

. . . . .

After you have pointed the given Number, ſeek what is the greateſt Cube Number in 314, the firſt Point, which, by the former little Table (*Page* 34) you will find to be 216, which is the neareſt that is leſs than 314, and its Root is 6; which put in the Quotient, and 216 under 314, and ſubtract it therefrom, and there remains 98, to which bring down the next Point, 432, and annex to 98, ſo will it make 98432 for the Reſolvend: then triple the Quotient 6, it makes 18, which write down, the Unit's Place, 8, under 3, the Tens Place of the Reſolvend: Then ſquare the Quotient 6, and triple that Square, and it makes 108, which write under the triple Quotient, one Place on the Left Hand; then add thoſe two Numbers together, and they make 1098 for the Diviſor: Then ſeek how often the Diviſor is contained in the Reſolvend (rejecting the Unit's Place thereof) that is, how often 1098 in 9843, which is eight times; put 8 in the Quotient, and the Cube thereof below the Diviſor, the Unit's Place under the Unit's Place of the Reſolvend: Then ſquare the 8 laſt put in the Qotient, and multiply 64, tqe Square thereof, by the triple Quotient 18, the Product is 1152; ſet this under the Cube of 8, the Units of this under the Tens of that.

Then multiply the triple Square of the Quotient by 8, the Figure laſt put in the Quotient, the Product is 864; ſet this down under the laſt Product, a Place more to the Left Hand: Then draw a Line under thoſe three, and add them together, and the Sum is 98432, which is called the Subtrahend, which being ſubtracted from the Reſolvend, the Remainder is nothing. Which ſhews the Number to be a true Cubick Number, whoſe Root is 68; that is, if 68 be cubed it will make 314432.

For, if 68 be multiplied by 68, the Product will be 4624, and this Product multiplied again by 68, the laſt Product is 314432; which ſhews the Work to be right.

```
                        68
                        68
                       ———
                       544
                      408
                      ————
   The Work.          4624
                        68
                     —————
                     36992
                    27744
                   ———————
         The Proof 314432
```

Example. Let the Cube Root of 5735339 be required.

After you have pointed the given Number, ſeek what is the greateſt Cube Number in 5, the firſt Point, which (by the little Table, *Page* 34) you will find to be 1, which place under 5, and 1, the Root thereof, in the Quotient, and ſubtract 1 from 5, and there remains 4; to which bring down the next Point, it makes 4735 for the Reſolvend: Then triple the 1, and it makes 3, and the Square of 1 is 1, and the Triple thereof is 3; which ſet one under another, in their Order, and added makes 33 for the Diviſor: Seek how often the Diviſor in the Reſolvend, and proceed as in the laſt *Example.*

5735339

. . . .
5735339(179 Root.
1

4735 Reſolvend.

3 Triple of the Quotient 1, the firſt Figure.
3 The triple Square of the Quotient 1.

33 The Diviſor.

343 The Cube of 7, the ſecond Figure of the Root.
147 The Square of 7, mult. in. the triple Quot. 3.
21 The triple Square of the Quotient, mult. by 7.

3913 The Subtrahend.

822339 The new Reſolvend.

51 The Trip. of the Quot. 17. the two firſt Fig.
867 The triple Square of the Quotient 17.

8721 Diviſor

729 The Cube of 9, the laſt Figure of the Root.
4131 The Square of 9, mult. by the trip. Quot. 51.
7803 The triple Square of the Quotient 867 by 9.

822339 The Subtrahend

. . . . . .

In this *Example*, 33, the firſt Diviſor, ſeems to be contained more than 7 times in 4735, the Reſolvend; but if you work with 9, or 8, you will find that the Subtrahend will be greater than the Reſolvend.

Some

Some more Examples for Practice.

. . .

32461759(319 The Root.

27

5461 Reſolvend.

9 The Triple of 3.

27 The triple Square of 3.

279 The Diviſor.

1 The Cube of 1. the ſecond Figure.

9 The triple Quotient by the Square of 1.

27 The triple Square mult. by 1, the ſecond Figure

2791 The Subtrahend

2670759 A new Reſolvend.

93 The Triple of 31.

2883 The triple Square of 31.

28923 The Diviſor.

729 The Cube of 9, the laſt Figure.

7533 The Square of 9, by 93, the triple Quotient

25947 The triple Square 2883 by 9.

2670759 The Subtrahend.

. . . . . .

. . .
84604519(439 The Root
64

20604 Reſolvend.

12 The triple of 4.
48 The triple Square of 4.

492 The Diviſor.

27 The Cube of 3.
108 The Square of 3, by the triple Quotient.
144 The triple Square by 3.

15507 The Subtrahend.

5097519 Reſolvend.

129 The triple of 43.
5547 The triple Square of 43.

55599 The Diviſor.

729 The Cube of 9.
10449 The Square of 9 by 129.
49923 The triple Square by 9.

5097519 The Subtrahend.

. . . .

· · ·
25969·989(638
216

43697 Reſolvend.

18 The Triple of 6.
108 The triple Square of 6.

1098 The Diviſor.

27 The Cube of 3, the ſecond Figure.
162 The Square of 3 by 18.
324 The triple Square 108 by 3.

34047 The Subtrahend.

9650989 Reſolvend.

189 The Triple of 63.
11907 The triple Square of 63.

119259 The Diviſor.

512 The Cube of 8.
12096 The Square of 8 by 189.
95256 The triple Square 11907 by 8.

9647072 The Subtrahend.

3917 The Remainder.

22069

. . . .
22069810125 (2805
8

14069 Reſolvend.

6 The Triple of 2.
12 Triple Square of 2.

126 The Diviſor.

512 The Cube of 8.
384 Square of 8 by 6.
96 Triple Square by 8.

13952 The Subtrahend.

117810125 New Reſolvend.

84 Triple of 28.
2352 Triple Squa. of 28.

23604 The Diviſor.

840 Triple of 280
235200 Triple Square of 280.

2352840 New Diviſor.

125 Cube of 5.
21000 Square of 5 by 840.
1176000 Triple Square by 5.

117810125 Subtrahend.

....

In this *Example*, 13952, being ſubtracted from the Reſolvend, 14069, the Remainder is 117, to which bring down 810, the third Point, & it makes 117810, for a new Reſolvend, and the next Diviſor is 23604, which you cannot have in the ſaid Reſolvend (the Unit's Place being rejected) ſo you muſt put 0 in the Quotient, and ſeek a new Diviſor; (after you have brought down your laſt Point to the Reſolvend) which new Diviſor is 2352840, which you will find to be contained five times. So proceed to finiſh the reſt of the Work.

25917056(295.9

17917 The Reſolvend.

6 The Triple of 2.
12 The triple Square of 2.

126 The Diviſor.

729 The Cube of 9, the ſecond Figure.
486 The Square of 9 by 6.
108 The triple Square by 9.

16389 The Subtrahend.

1528056 The Reſolvend.

87 The Triple of 29.
2523 The triple Square of 29.

25317 The Diviſor.

125 Cube of 5, the 3d Fig.
2175 The Square of 5 by 87.
12615 The triple Square by 5.

1283375 The Subtrahend.

244681000 The Reſolvend.

885 The Triple of 295.
261075 The triple Square of 295.

2611635 The Diviſor.

729 The Cube of 9, the laſt Figure.
71685 The Square of 9 by 885.
2349675 The triple Square by 9.

235685079 The Subtrahend.

8995921 The Remainder.

In this *Example* I annex three Cyphers to the Remainder, which makes the 3d Reſolvend by which Means I bring one Place of Decimals. And ſo you may proceed to more Decimal Places at Pleaſure, by annexing 3 Cyphers to the next Remainder, and carrying on the Work as before,

93759.575070(45.42
64

29759 The Reſolvend.

12 Triple of 4, the firſt Figure.
48 Triple Square of 4.

492 Diviſor.

125 Cube of 5, the ſecond Figure.
300 The Square of 5, by 12, the triple Quotient.
240 The triple Square by 5.

27125 The Subtrahend.

2634575 The Reſolvend.

135 The Triple of 45.
6075 The triple Square of 45.

60885 The Diviſor.

64 The Cube of 4.
2160 The Square of 4 by 135.
24300 The triple Square by 4.

2451664 The Subtrahend.

182911070 The Reſolvend.

1362 The Triple of 45.4.
618348 The triple Square of 45.4.

6184842 The Diviſor.

8 The Cube of 2.
5448 The Square of 2 by 1362.
1236696 The triple Square by 2.

123724088 The Subtrahend.

59186982 The Remainder.

In extracting the Cube Root of a mixed Number, always obſerve to make the Decimal Part to conſiſt of either, three, ſix, nine, *&c.* Places; that is, always to conſiſt of even Points, as in the laſt *Example*, where the Decimal Places were five, to which I annexed a Cypher, to make it up ſix, and ſo I proceed to point it; and by that means I have a Point falls upon the Unit's Place of whole Numbers; which you muſt always obſerve.

To extract the CUBE ROOT *out of a Fraction.*

This is the ſame to do as in whole Numbers, obſerve but the foregoing Directions for the true pointing thereof; for, as was before directed, the Decimal muſt always conſiſt of three, ſix, nine, *&c.* Places; and if it be not ſo it muſt be made ſo by annexing of Cyphers, as is above ſaid.

If the Cube Root of a Vulgar Fraction be required, you muſt firſt reduce it to a Decimal, and then extract the Root thereof.

Examples of each follow.

Example 1. Let the Cube Root of .401719179 be requir'd.

.401719179(.737 Root.
343 Cube of 7.

58719 Reſolvend.

21 Triple of 7.
147 Triple Square of 7.

1491 Diviſor.

27 Cube of 3.
189 Square of 3 by 21.
441 Triple Square by 3.

56017 Subtrahend.

12702179 Reſolvend.

219 Triple of 73.
15987 Triple Square of 73.

160089 Diviſor.

343 Cube of 7.
10731 Square of 7 by 219.
111909 Triple Square by 7.

11298553

1403626 Remainder.

Example 2. Let the Cube Root of .0001416 be required.

. . .
.000141600(.052 Root.
125

16600 Reſolvend.

15 The Triple of 5.
75 The triple Square of 5.

765 The Diviſor.

8 The Cube of 2.
60 The Square of 2 by 15.
150 The triple Square by 2.

15608 The Subtrahend.

992 Remainder.

Example 3. Let $\frac{5}{276}$ be a Vulgar Fraction whoſe Cube Root is required.

By the firſt Rule of Chapter II. reduce the Vulgar Fraction to a Decimal.

276)5.000000000(.018115942
. . . .

2240
320
440
1640
2600
1160
560

8

.018115942

$.01\dot{8}11\dot{5}94\dot{2}($.262 Root
8

10115 Reſolvend.

6 The Triple of 2.
12 The triple Square of 2.

126 The Diviſor.

216 The Cube of 6.
216 Square of 6 by the Triple of 2
72 The triple Square by 6.

9576 The Subtrahend.

539942 Reſolvend.

78 The Triple of 26.
2028 The triple Square of 26.

20358 The Diviſor.

8 The Cube of 2.
312 The Square of 2 by 78.
4056 Triple Square 2028 by 2.

408728 The Subtrahend.

131214 Remainder.

You may prove the Truth of the Work by cubing the Root found, as was ſhewed in the firſt *Example* ; and if any Thing remains, add it to the ſaid Cube, and the Sum will be the given Number, if the Work is rightly performed.

I will ſhew the Proof of the fiſth *Example*, (*Page* 48) the given Number being 259697989, whoſe Root is 638, it being a Surd Number there remains 3917.

	638
	638
	5104
	1914
	3828
The Square	407044
	638
	3256352
	1221132
	2442264
The Cube	259694072
The Remainder add	3917
Proof equal to the given Number	259697989

EXTRACTION *of* ROOTS, *out of any Single Power, by one* General Method, *deduced from the* CONVERGING SERIES.

In order to perform this obſerve carefully the following

GENERAL RULES.

Rule 1.] Point the given Number as directed p. 32, 41, *&c.*

Rule 2.] Find the neareſt Root to the firſt Period towards the Left Hand, and to this Root annex a Cypher for every Period, except one, in the given Number, and raiſe it to the Height of the given Power.

Rule 3.] Take the Difference between this involved Root and this given Number for a Dividend, which *Note* with the Sign + if the given Number be greater, but with — if the given Number was leſs than the involved Root.

Rule 4.] Raiſe the ſaid Root and Cyphers to a Power whoſe Index is one leſs than the Index of the given Power, and multiply this Number by the Index of the given Power for a Diviſor, by which divide the Dividend, taking but one Figure into the Quotient.

Rule 5.] If the aforeſaid Difference was noted with +, this Quotient Figure muſt be annexed to the firſt taken Root, but if noted with — it muſt be taken from the ſaid Root, and the Sum or Difference muſt be wrought with in every Reſpect as with the firſt taken Root, and the next Quotient you get, you take two Figures, and the next Quotient after that you may take three or four Figures, *&c.* every time nearly, doubling the Number of Figures, by which means you may have the Root of any ſingle Power to a great many Places of Figures, which compared with the old Method is vaſtly eaſy: We ſhall examine this Method by an *Example* or two of the *Square Root,* and then apply it to the Cube, or higher Powers.

Example 4. What is the Square Root of 321489?

OPERATION.

360000=Square of 600.

321489 given Number Subtract.
1200) 38511 (600 firſt Root.
36000 30 Quotient ſubtracted.
Remainder 251 560 ſecond Root.
321489 given Number.
313600 Square of 560 Subtract.
1120) +7889 (560 ſecond Root.
7840 7.0 Quotient add.
Remainder 49.0 567.0 third Root.

Here the neareſt Root to the firſt Period 32 is 6, and there are two Periods more in the given Number ſo I put two Cyphers to 6 and it makes 600, which multiplied by itſelf, gives 360000, which being greater than 321489, I ſubtract 321489 therefrom, and ſet the Sign—before the Remainder or Dividend 38511. Now the Index of the Square being 2, and the next leſs Power to the Square being 1, or the firſt Power, (*Rule* 4.) the double of 600, *viz.* 1200, is the firſt Diviſor, and the Quotient comes out 32, but we are to take but the firſt Figure 3, ſo I put a Cypher in the Room of 2; and here *Note*, the Reaſon of taking but one Figure in the firſt Diviſor, two Figures in the ſecond Diviſor, *&c*, is, becauſe if we take more, the Root will be too much. Hence, when the Quotient is to be ſubtracted you may carry one to the firſt Figure towards the Right Hand, ſo in this *Example*, inſtead of taking 30 from 600, I take 40 therefrom; for it is to be ſubtracted, becauſe 38511 is noted with —, the carrying of this 1, would ariſe from taking the 2 out of 0, *viz.* 32 from 600, but when the Quotient is to be added, ſuch Increaſe muſt not be regarded.

But 40 taken from 600 leaves 560 for the ſecond Root, which gives 313600 when ſquared, and being leſs than the given Number, is ſubtracted therefrom, and + 7889 remains for a ſecond Dividend, which divided by twice 560, *viz.* 1120, gives 7.0; for here we are to take two Figures into the Quotient, and becauſe the Dividend is *noted* with + this 7.0 is added to 560, which gives 567 for the third Root, and being ſquared is equal to the given Number; ſo 567 is the true ſquare Root of 321489.

Example 5. What is the Square Root of 7?

OPERATION.

9 the neareſt Square to 7,
7 the given Number,

6)— 2.0 { 3 the firſt Root,
18 { 0.3 the Quotient ſubtract,

Remainder (2) 2.6 ſecond Root,

7 given Number,
6.76 Square of the ſecond Root,

52)+.240(2.6 ſecond Root,
208 .046 Quotient add,

320 2.646 third Root,
312

(8) Remainder.

7.001316 Square of 2.646,
7 ——— given Number ſubtract,
5.292)—.0013160 { 2.646 third Root,
10584 { .0002486 Quot. ſubtract,

25760 2.6457513 fourth.
21168

45920 } Root true to the laſt
42336 } Figure.

35840
31752

Remains (4088)

Here the neareſt Squares to the given Number 7, are 4 and 9, but 9 is nearer to 7 than 4, ſo I take 9, whoſe Square Root is 3, and twice 3 is 6, the firſt Diviſor; alſo becauſe the Quotient 0.3 is to be ſubtracted from the firſt Root 3, I ſubtract 0.4 as directed in the laſt *Example*; for the ſame Reaſon I ſubtract .0002487, inſtead of .0002486 from the the third Root 2. 646, and ſo get 2.6457513 for the Square Root of 7, and if you ſquare this laſt Root and proceed as before, you may rely on ſeven or eight Figures in the Quotient, and ſo have the Root to fourteen or fifteen Places of Decimals.

This Method of extracting Roots proves itſelf as it goes on, for here 3 is more than the Square Root of 7, but 2.6 is ſomewhat leſs; alſo 2.646 is ſomewhat more, and again 2.6457513 is ſomewhat too little; ſo that every one of theſe Roots come nearer the Truth than the preceding one; that is, 2.6 is nearer than 3, but 2.646 is nearer than either 3, or 2.6, *&c.*

Example 6. What is the Cube Root of 1728?

. . OPERATION.
1728 the given Number,
1000 the Cube of 10 ſubtract,
300) + 728 (10 firſt Root,
600 2 Quotient add,
—— ——
Remains 128 12 ſecond Root.

Here the Cube Root of 1, the firſt Period, is 1, to which annex a Cypher, becauſe there is another Period in the propoſed Number 1728, and it becomes 10, whoſe Cube 1000, taken from 1728, leaves 728 for a Dividend; now the next Power below the Cube is the Square, and the Index of the Cube is 3, therefore by the fourth Rule, the Square of 10 is 100, which multiplied by 3, gives 300 for the firſt Diviſor, and the Quotient 2 added to 10 gives 12 for the ſecond Root, which cubed gives 1728; hence 12 is the exact Cube Root of 1728.

Example 7. Let it be required to extract the Cube Root out of

.067507824239 given Num.
64 Cube of 4

Three times the Square of .4 = } 48)+.003507324239 } .4 firſt Root,
336 } .007 Quo. add

Remains 14 .407 ſec. Root

.067507824239 given Numb.
.067419143 Sub.C.of.407 2d R.

3 times the Square of .407 is } .496947)+.000088681239 } .407 2d R.
496947 } .000178 Q. add

3898653 .407178 (third
3478629 } Root to the laſt Figure.

4200249
3975576

Remains 224673

N O T E, Becauſe the firſt Figure that ariſes in the firſt Quotient is 0, you may in ſuch Caſes take two Figures into the Quotient, ſo here I take in 07, and becauſe two Figures are taken into this Quotient, you may take three or four Figures into the next Quotient, as *per* Work.

So that the Cube Root of .067507824239 is .4; or nearer .407; or nearer .407178.

Example 8. What is the Cube Root of 282? And of 231?

Operation I.

343 Cube of 7,
282 given Number,

147)—61.0 } 7 firſt Root,
58.8 } 0.4 Quotient ſubtract,

Remains 22 6.5 ſecond Root,

282 given Number,
274.625 Cube of 6.5

126.75)+7.3750 } 6.5 ſec. Root,
6.3375 } .058 Quotient add,

103750 6.558 third Root.
101400

Remains 2350

Operation II.

216
231

108)+150 } 6
108 } 0.1
Remains 42 6.1

231
226.981
111.63)+4.0190 } 6.1
33489 } .036
67010 6.136
66978
Remains 32

So the Cube Root of 282 is 6.558, And the Cube Root of 231 is 61.36 nearly.

Example 9. What is the Cube Root of 2150.42?

OPERATION.

2150.42	given Number,
1000	Cube of 10,
300)+1150.42	10 firſt Root,
900	3 Quotient add.
Remains 250.42	13 ſecond Root,
2197	Cube of 13,
2150.42	given Number,
1521)—46.58	13 ſecond Root
4563	0.0306 Quot. ſubtr.
9500	12.9693 third Root.
9126	
Remains (374)	

Example 10. What is the Cube Root of $1\frac{1}{3}$?

OPERATION.

The Decimal of $1\frac{1}{3}$ is 1. 3333 given Number,
1 Cube of

3) 0.3333 { 1 firſt Root,
.3 { 0.1 Quo. add

Remains .033 &c. 1.1 ſec. Root.

1.3333 given Number,
1.331 Cube of 1.1

3.63)+.002333 { 1.1 ſecond Root,
2178 { .0006427

1553 1.1006427
1452

1013
726

2873
2541

Remains (332)

In *Examſle* 9. Becauſe the firſt Figure in the ſecond Quotient is a Cypher, I take four Figures into that Quotient, and ſo get the Cube Root of 2150.42 to be 12.9693, but in *Example* 10, the two firſt Figures in the ſecond Quotient are Cyphers, *viz.* the Places of Hundreds and Thouſands are each a Cypher, and therefore in this Caſe, I take Figures into the ſaid Quotient (you may take ſeven or eight Figures if you pleaſe) and ſo get 1.1006327 for the Cube Root of $1\frac{1}{3}$, or 1.333333333, &c,

CHAP.

CHAP. IX.

Multiplication of Feet, Inches and Parts.

IN the multiplying of Feet, Inches, *&c*, I ſhall endeavour to lay down ſuch eaſy and familiar Rules, as may eaſily be underſtood by the meaneſt Capacity.

Example 1. Let 7 Feet 9 Inches be multiplied by 3 Feet 6 Inches.

F.	I.	
7	9	
3	6	
23	3	Prs.
3	10	6
27	1	6

Firſt, multiply 9 Inches by 3, ſaying, 3 times 9 is 27 Inches, which make 2 Feet 3 Inches; ſet down 3 under Inches, and carry 2 to the Feet, ſaying, 3 times 7 is 21, and 2 that I carry make 23; ſet down 23 under the Feet.

Then begin with 6 Inches, ſaying 6 times 9 is 54 Parts, which is 4 Inches and 6 Parts; ſet down 6 Parts, and carry 4, ſaying, 6 times 7 is 42, and 4 that I carry is 46 Inches, which is 3 Feet 10 Inches; which ſet down, and add all up together, and the Product is 27 Feet, 1 Inch, 6 Parts.

Example 2. Let 75 Feet 7 Inches be multiplied by 9 Feet 8 Inches.

F.	I.	
75	7	
9	8	
680	3	
50	4	8
730	7	8

Firſt, multiply by 9 Feet, ſaying, 9 times 7 is 63, which is 5 Feet 3 Inches; ſet down 3 and carry 5, ſaying 9 times 5 is 45, and 5 that I carry is 50; ſet down 0 and carry 5, ſaying, 9 times 7 is 63, and 5 is 68; ſet down 68, and proceed to multiply by 8 Inches, ſaying, 8 times 7 is 56, the Twelves in 56 are 4 times, and 8 remain; ſet 8 a Place to the Right Hand, and carry 4; Then multiply 75 by 8, and the Product is 600, and 4 that I carry is 604, which divided by 12, the Quotient is 50 Feet, and 4 remains; ſet down 50 Feet, 4 Inches, and add all up together, and you will find the Product 730 Feet, 7 Inches, 8 Parts.

I will repeat the laſt *Example* again, and ſhew another Way to work it, which I think is better and more expeditious when there are more Figures than one in the Feet: Thus,

F.	I.	P.
75	7	
9	8	
680	3	
25	2	4$\frac{1}{3}$
25	2	4$\frac{1}{3}$
730	7	8

Multiply by 9 Feet firſt, as above directed; then, inſtead of multiplying by 8 Inches, let the 8 Inches be parted into ſuch aliquot or even Parts of a Foot as you find to be contained in that Figure; if you take ſuch Parts of the Multiplicand, and add them to the former Product, the Sum will give the Anſwer: Thus, 8 Inches may be parted into 4 and 4, becauſe 4 is the third Part of 12: So, if you take the third Part of 75 Feet, 7 Inches, and ſet it down twice, and add all together, the Sum will be 730 Feet, 7 Inches, 8 Parts, the ſame as before: Thus, ſay how often 3 in 7, which is twice, ſet down 2; then, becauſe twice 3 is 6, ſay, 6 out of 7 and there remains 1, for which you muſt add 10 to the 5, and it makes 15; then the Threes in 15 are 5 times, ſet down 5; and, becauſe 3 times 5 is 15, there is 0 remains. Then go to the 7 Inches, ſaying, the Threes in 7 are twice, ſet down 2 in the Inches; and, becauſe twice 3 is 6, take 6 out of 7 and there remains 1 Inch, which is 12 Parts; then the Threes in 12 are 4 times, and 0 remains. So the third Part of 75 Feet 7 Inches, is 25

Feet, 2 Inches, 4 Parts; which ſet down again, and add all together, the Sum is 730 Feet, 7 Inches, 8 Parts, the ſame as before.

Example 3. Let 97 Feet, 8 Inches, be multiplied by 8 Feet, 9 Inches.

F.	I.	
97	8	$\frac{1}{2}$
8	9	$\frac{1}{4}$
781	4	
48	10	
24	5	
854	7	

Begin, firſt to multiply by 8 Feet, ſaying, 8 times 8 is 64 Inches, that is, 5 Feet 4 Inches; ſet down 4 Inches, and carry 5, ſaying, 8 times 7 is 56, and 5 I carry is 61; ſet down 1, and carry 6, ſaying, 8 times 9 is 72, and 6 I carry is 78, which ſet down: Then, inſtead of multiplying by 9 Inches, take the aliquot Parts of 12, which 9 makes, which is 6 and 3, 6 Inches being half 12, and 3 the fourth Part; therefore take the half of 97 Feet, 8 Inches, which is 48 Feet, 10 Inches, and becauſe 3 is half 6, you may take the half of 48 Feet, 10 Inches, which is 24 Feet, 5 Inches; add all up together, and the Sum is 854 Feet, 7 Inches. See the Work, as above.

Example 4. Let 75 Feet, 9 Inches, be multiplied by 17 Feet 7 Inches.

F.	I.	
75	9	
17	7	
525		
75		
25	3	P.
18	11	3
8	6	
4	3	
1331	11	3

I this *Example*, becauſe there are more than 12 Feet in the Multiplier, therefore I firſt multiply the 75 by 17 Feet; then becauſe the aliquot Parts in 7 Inches are 4 and 3, that is, a Third and a Fourth, I take the third Part of 75 Feet 9 Inches, which is 25 Feet, 3 Inches, and the fourth Part thereof is 18 Feet, 11 Inches, 3 Parts; then the aliquot Parts of 9 Inches are 6 and 3, that is, half and a fourth; therefore I take half 17 Feet, which is 8 Feet, 6 Inches, and the fourth Part is 4 Feet, 3 Inches, not meddling with the 7 Inches, becauſe that was multiplied into the 9 before) then add all theſe together, and the Sum is 1331 Feet, 11 Inches, 3 Parts.

Example 5. Let 87 Feet, 5 Inches, be multiplied by 35 Feet, 8 Inches.

F.	I.		
87	5		
35	8		
435			
261		P.	
29	1	8	$\frac{1}{3}$
29	1	8	$\frac{1}{3}$
11	8	0	$\frac{1}{3}$
2	11	0	$\frac{1}{12}$
3117	10	4	

Work here as in the laſt *Example*. After you have multiplied the Feet, then take the aliquot Parts of 8 Inches, which is two Thirds; therefore take the third Part of 87 Feet 5 Inches, and ſet it down twice; thus, the third Part of 87 Feet, 5 Inches, is 29 Feet 1 Inch, 8 Parts; ſet this down twice: Then the aliquot Parts of 5 Inches are 4 and 1, that is, a third Part and a twelfth Part; therefore take a third Part of 35, which is 11 Feet, 8 Inches, and a twelfth Part of 35, is 2 Feet, 11 Inches; ſet all theſe one under another, and add them together, and the Sum is 3117 Feet, 10 Inches, 4 Parts.

Example 6. Let 259 Feet, 2 Inches, be multiplied by 48 Feet, 11 Inches.

F.	I.					
259	2					
48	11					
2072						
1036						
129	7	P.	$\frac{1}{2}$			
86	4	8	$\frac{1}{3}$			
21	7	2	$\frac{1}{12}$		F.	I.
8	1	10	$\frac{1}{6}$	of	48	11
12677	8	8				

Firſt, multiply the Feet; then take the aliquot Parts of 11, which will be 6, 4 and 1, that is, an Half, a Third, and a Twelfth; therefore take the Half of 259 Feet 2 Inches, which is 129 Feet 7 Inches, and a third Part is 86 Feet, 4 Inches 8 Parts, and the twelfth Part of 259 Feet, 2 Inches, is 21 Feet, 7 Inches, 2 Parts; or (becauſe 1 is the fourth Part of 4) you may more readily take the fourth Part of 86 Feet, 4 Inches, 8 Parts, which is alſo 21 Feet, 7 Inches, 2 Parts; then becauſe 2 Inches (in the Multiplicand) is one Sixth of 12, take the ſixth Part of the Feet, only, in the Multiplier, *viz.* 48, which quotes 8 Feet, and add all together, and the Sum is 12677 Feet, 6 Inches, 10 Parts. See the foregoing Work corrected.

I ſhall ſet down only the working of ſome few *Examples* in Feet and Inches, and then proceed to multiply Feet, Inches, and Parts, *&c.*

	F.	I.	P.		
	179	3			
	38	10			
	1432				
	537				
	89	7	6	$\frac{1}{2}$	
	59	9	0	$\frac{1}{3}$	F.
	9	6	0	$\frac{1}{4}$	*of* 38
Product	6960	10	6		

	F.	I.	P.		
	246	7			
	46	4			
	1476				
	984				
	82	2	4	$\frac{1}{3}$	F.
	15	4	0	$\frac{1}{3}$	*of* 46
	11	6	0	$\frac{1}{4}$	
Prod.	11425	0	4		

	F.	I.	P.		
	246	7			
	36	9			
	1476				
	738				
	123	3	6	$\frac{1}{2}$	
	61	7	9	$\frac{1}{4}$	F.
	12	0	0	$\frac{1}{3}$	*of* 36
	9	0	0	$\frac{1}{4}$	
Prod.	9061	11	3		

	F.	I.	P.		
	257	9			
	39	11			
	2313				
	771				
	128	10	6	$\frac{1}{2}$	
	85	11	0	$\frac{1}{3}$	
	21	5	9	$\frac{1}{12}$	F.
	19	6	0	$\frac{1}{2}$	*of* 39
	9	9	0	$\frac{1}{4}$	
Prod.	10288	6	3		

Example 11. Let 7 Feet, 5 Inches, 9 Parts, be multiplied by 3 Feet, 5 Inches, 3 Parts.

F.	I.	P.		
7	5	9		
3	5	3		
22	5	3	S.	
3	1	4	9	T.
	1	10	5	3
25	8	6	2	3

In this *Example*, I firſt begin with 3 Feet, and thereby multiply 7 Feet, 5 Inches, 9 Parts: Firſt I ſay, 3 times 9 is 27 Parts, that this, 2 Inches and 3 Parts, ſet down 3 under the Parts, and carry 2, ſaying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Foot 5 Inches; ſet down 5 Inches and carry 1, and ſay, 3 times 7 is 21, and 1 I carry is 22; ſet down 22 Feet. Then begin with 5 Inches, ſaying, 5 times 9 is 45, which is 45 Seconds, which makes 3 Parts and 9 Seconds; ſet down 9 Seconds a Place towards the Right Hand, and carry 3 Parts, ſaying, 5 times 5 is 25, and 3 I carry is 28, which is 2 Inches and 4 Parts; ſet down 4 Parts, and carry 2, ſaying, 5 times 7 is 35, and 2 I carry is 37, which is 3 Feet 1 Inch; ſet down 3 Feet 1 Inch, and begin to multiply by 3 Parts, ſaying, 3 times 9 is 27 Thirds, that is, 2 Seconds and 3 Thirds; ſet down 3 Thirds, and carry 2, ſaying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Part and 5 Seconds; ſet down 5 Seconds, and carry 1, ſaying, 3 times 7 is 21, and 1 I carry is 22, which is 1 Inch and 10 Parts; which ſet down, and add all up, and the Product is 25 Feet, 8 Inches, 6 Parts, 2 Seconds, 3 Thirds.

NOTE, That in multiplying Feet, Inches and Parts, *&c.* if Feet be multiplied by Feet, the Product is Feet; and Feet multiplied by Inches, the Product is Inches, and the twelfth Part is Feet; and Parts multiplied by Feet, the Product is Parts, and the twelfth Part thereof is Inches; Parts multiplied by Inches, the Product is Seconds, and the twelfth Part thereof is Parts; and Parts multiplied by Parts, the Product is Thirds, and the twelfth Part thereof is Seconds.

So that if you begin to multiply Parts by Feet in the firſt Row, and Parts by Inches in the ſecond Row, and Parts by Parts in the third Row, the firſt Figure in every Row will ſtand a Place more towards the Right Hand, as you may ſee in the laſt *Example.*

Example 12. Let 37 Feet, 7 Inches, 5 Parts, be multiplied by 4 Feet, 8 Inches, 6 Parts.

F.	I.	P.			
37	7	5			
4	8	6			
150	5	8	S.		
12	6	5	8		$\frac{1}{3}$
12	6	5	8	T.	$\frac{1}{3}$
1	6	9	8	6	$\frac{1}{2}$
177	1	5	0	6	

Firſt, I mnltiply by 4 Feet, ſaying, 4 times 5 is 20, which is 1 Inch, 8 Parts; ſet down 8, and carry 1, ſaying, 4 times 7 is 28, and 1 I carry is 29, which is 2 Feet, 5 Inches; ſet down 5 Inches, and carry 2, ſaying, 4 times 7 is 28 and 2 I carry is 30; ſet down 0, and carry 3, and ſay, 4 times 3 is 12, and 3 is 15; ſet down 15. Then I begin with 8 Inches; but becauſe the Feet in the Multiplicand are more than 12, it will be the beſt Way to work for the aliquot Parts of 8; ſo here I work for 4 Inches, and ſet that down twice, 4 being the third Part of 12; therefore take the third Part of 37 Feet, 7 Inches, 5 Parts, which is 12 Feet, 6 Inches, 5 Parts, 8 Seconds; ſet this down twice. Then begin with 6 Parts; but, inſtead of multiplying, take half 37 Feet, 7 Inches, 5 Parts; [becauſe 6 is half 12) and ſet it a Place more to the Right Hand: Thus, the half of 37 Feet is 18, which I muſt count 18 Inches, becauſe the Multiplier is 6 Parts; ſo the half of 37 Feet 7 Inches, 5 Parts, is 1 Foot, 6 Inches, 9 Parts, 8 Seconds, 6 Thirds; which ſet down, and add all up together, and the Sum is 177 Feet, 1 Inch, 5 Parts, 0 Seconds, 6 Thirds.

Example 13. Let 311 Feet, 4 Inches, 7 Parts, be multiplied by 36 Feet, 7 Inches, 5 Parts.

F.	I.	P.			
311	4	7			
36	7	5			
1866					
933			S.		
103	9	6	4		1/3
77	10	1	9	T.	1/4
8	7	9	6	4	1/3
2	1	11	4	7	1/4
12	0	0	0	0	1/3
1	0	0	0	0	1/12
	9	0	0	0	1/3
11492	2	4	11	11	

In this *Example*, becauſe the Feet both in the Multiplier and Multiplicand are compound Numbers, I firſt multiply the Feet one by the other: then take the aliquot Parts of 7 Inches, which are 4 Inches and 3, that is, a third and a fourth Part; ſo take the third Part of 311 Feet 4 Inches, 7 Parts, which is 103 Feet, 9 Inches, 6 Parts, 4 Seconds, and the fourth Part is 77 Feet, 10 Inches, 1 Part, 9 Seconds; ſet theſe down one under another, the Feet under the other Feet; then the aliquot Parts of 5 Parts are 4 and 1, that is, a third and a twelfth Part; ſo the third Part of 311 Feet, 4 Inches, 7 Parts, is 103 Feet, 9 Inches, 6 Parts, 4 Seconds; but becauſe the Multiplier is Parts, it muſt be ſet a Place to the Right Hand, that is, the 103 muſt be Inches, which is 8 Feet, 7 Inches; therefore I ſet down 8 Feet, 7 Inches, 9 Parts, 6 Seconds, 4 Thirds.

Then, becauſe 1 Inch is a fourth Part of 4 Inches, therefore I take a fourth Part of 8 Feet, 7 Inches, 9 Parts, 6 Seconds, 4 Thirds, which is 2 Feet, 1 Inch, 11 Parts, 4 Seconds, 7 Thirds; which is the ſame as if I had taken a twelfth Part of 311 Feet, 4 Inches, 7 Parts. Then, for 4 Inches in the Multiplicand, inſtead of multiplying 36 Feet by it, take a third Part, becauſe 4 Inches is a third Part of 12; ſo the third Part of 36 is 12 Feet, and the aliquot Parts of 7 Parts are 4 and 3, that is, a Third and a Fourth; ſo the third Part of 36 is 12, which now is 12 Inches, that is, 1 Foot, and the fourth Part is 9 Inches; add all theſe together, and the Sum will be 11402 Feet, 2 Inches, 4 Parts, 11 Seconds 11 Thirds.

Example 14. Let 8 Feet, 4 Inches, 3 Parts, 5 Seconds, 6 Thirds, be multiplied by 3 Feet, 3 Inches, 7 Parts, 8 Seconds, 2 Thirds.

	F.	I.	P.	S.	T.				
	8	4	3	5	6				
	3	3	7	8	2				
	25	0	10	4	6				
	2	1	0	10	4	6			
		4	10	6	0	2	6		
			5	6	10	3	8	0	
				1	4	8	6	11	0
Product	27	7	3	5	1	8	8	11	0

In this laſt *Example* there is no Difficulty, if you do but obſerve the former Directions, and ſet every Row a Place more to the Right Hand.

I ſhall only ſet down the Working of ſome few *Examples* more, and ſo conclude this Chapter.

F.	I.	P.		
321	7	3		
9	3	6		
2894	5	3	S.	
80	4	9	9	T.
13	4	9	7	6
2988	2	10	4	6

F.	I.	P.		
42	7	8		
7	3	6		
298	5	8		
10	7	11	S.	
1	9	3	10	
310	10	10	10	

F.	I.	P.		
124	7	9		
14	6	2		
496				
124			S.	
62	3	10	6	T.
1	8	9	3	6
7	0	0	0	0
1	2	0	0	0
	7	0	0	0
	3	6	0	0
1809	1	1	9	6

F.	I.	P.		
259	10	8		
18	5	4		
2072				
259			S.	
86	7	6	8	
21	7	10	8	T.
7	2	7	6	8
9	0	0	0	0
6	0	0	0	0
1	0	0	0	0
4793	6	0	10	8

267

F.	I.	P.		
267	7	10		
25	9	7		
1335				
534				
133	9	11	S.	
66	10	11	6	
11	1	9	11	T.
1	10	3	7	10
12	6	0	0	0
2	1	0	0	0
1	0	6	0	0
	8	1	0	0
6905	0	7	0	10

F.	I.	P.		
317	9	7		
37	5	9		
2219				
951			S.	
105	11	2	4	
26	5	9	7	T.
13	2	10	9	6
6	7	5	4	9
18	6	0	0	0
9	3	0	0	0
1	6	6	0	0
	3	1	0	0
11910	9	11	1	3

The RULE at large.

Feet multiplied by	*Feet* *Inches* *Parts*	*produce*	*Feet.* *Inches.* *Parts.*
Inches multiplied by	*Feet* *Inches* *Parts*	*produce*	*Inches.* *Parts.* *Seconds.*
Parts multiplied by	*Feet* *Inches* *Parts*	*produce*	*Parts.* *Seconds.* *Thirds*

N. B. *Twelve Thirds make one Second, Twelve Seconds make one Part, Twelve Parts make one Inch, and Twelve Inches make one Foot, in this Multiplication of Feet, Inches and Parts.*

THE

The Complete Meaſurer.

PART II.

CHAP. I.

Menſuration of SUPERFICIES.

Superficial Figures are all ſuch, as have only Length and Breadth, not having any commenſurable Thickneſs.

---

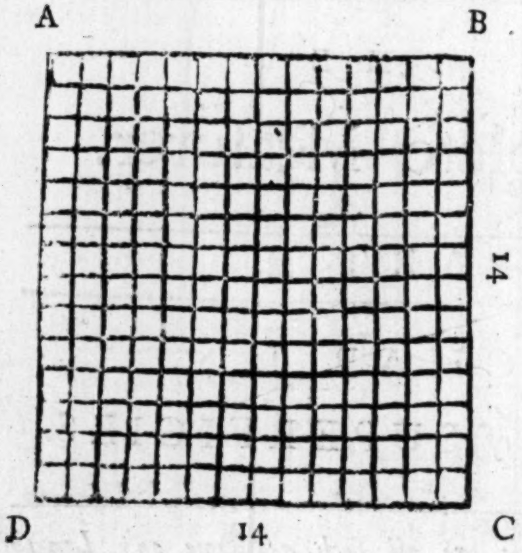
§. 1. *Of a* SQUARE.

A SQUARE is a Geometrical Figure, having four equal Sides, and as many right (or ſquare) Angles. To find the ſuperficial Content thereof, this is

The RULE.

Multiply the Side into itſelf, and the Product is the Content.

Let ABCD be a Geometrical Square given, each Side being 14 Feet, Yards, Poles, or other Meaſure; multiply 14 by itſelf, and the Product is 196, which is the ſuperficial Content.



14
14
56
14
196 Product.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1, in the Line of Numbers, to 14; the ſame Extent will reach from the ſame Point turned forward to 196.

DEMONSTRATION

Let each Side of the given Square be divided into 14 equal Parts, and Lines drawn from one another, croſſing each other within the Square; ſo ſhall the whole great Square be divided into 196 little Squares, as you may ſee in the Figure above, equal to the Number of ſquare Feet, Yards, or Poles, or other Meaſures, by which the Side was meaſured.

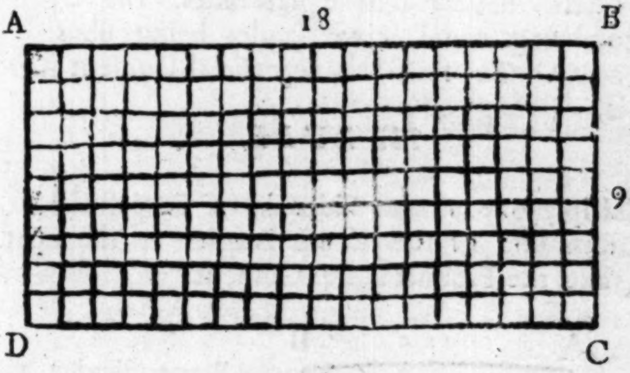
§ II.

§ II. *Of a* PARALLELOGRAM, *or* LONG SQUARE.

A Parallelogram is a Figure having four Sides, and as many right Angles, the oppoſite Sides thereof being equal and parallel. To find the ſuperficial Content thereof. this is

The RULE.

Multiply the Length by the Breadth, and the Product is the ſuperficial Content.



Length 18
Breadth 9

Product 162

Let ABCD be a long Square, the Length thereof 18 Feet, and the Breadth 9 Feet, which multiplied together, the Product is 162. the ſuperficial Content thereof.

By Scale *and* Compaſſes.

Extend the Compaſſes in the Line of Numbers from 1 to 9, the ſame Extent will reach from 18 down to 162, the ſquare Feet.

DEMONSTRATION.

If the Sides AB and CD, be each divided into 18 equal Parts, repreſenting 18 Feet, and the Lines AD and BC each divided in 9 equal Parts, and Lines drawn from Point to Point, croſſing each other within the Figure; thoſe Lines will make thereby ſo many little Squares as there are ſquare Feet, *viz.* 162.

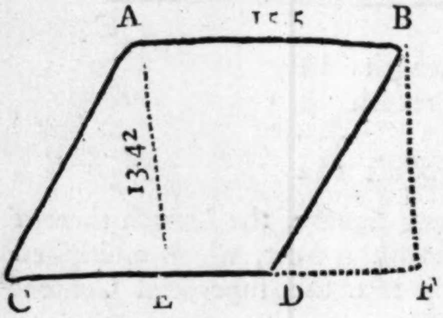
---

§ III. *Of a* RHOMBUS.

A Rhombus is a Figure repreſenting a Quarry of Glaſs, having four equal Sides, the Oppoſites thereof being equal, two Angles being obtuſe, and two acute. To find the ſuperficial Content thereof, this is,

The RULE.

Multiply one of the Sides by a Perpendicular let fall from one of the obtuſe Angles to the oppoſite Side, and the Product is the Content.



Perpendicular 13.42
The Side 15.5

6710
6710
1342

Product 208.010

Let ABCD be a Rhombus given, whoſe Sides are each 15.5 Feet, and the Perpendicular EA is 13.42, which multiplied together, the Product is 208.01; which is the ſuperficial Content of the Rhombus, that is, 208 Feet, and one hundredth Part of a Foot.

By Scale *and* Compaſſes,

Extend the Compaſſes from 1 to 13.42; that Extent will reach from 15.5, the ſame Way to 208 Feet, the Content.

DEMONSTRATION.

Let CD be extended out to F, making DF equal to CE, and draw the Line BF; ſo ſhall the Triangle DBF be equal to the Triangle ACE: For DF and CE are equal, and BF is equal to AE, becauſe AB and CF are parallel. Therefore the Parallelogram ABEF is equal to the Rhombus ABCD.

---

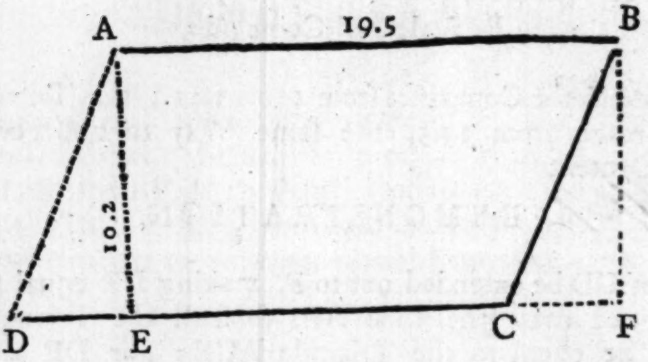
§ IV. *Of a* RHOMBOIDES.

A Rhomboides is a Figure having four Sides, the oppoſite whereof are equal and parallel; and alſo four Angles, the oppoſite whereof are equal. To find the ſuperficial Content thereof, this is

The RULE

Multiply one of the longeſt Sides thereof by the Perpendicular let fall from one of the obtuſe Angles to one of the longeſt Sides, and the Product is the Content.

```
  19.5
  10.2
 ------
   390
 1950
 ------
 198.90
```



Let ABCD be a Rhomboides given, whoſe longeſt Side AB or CD is 19.5 Feet, and the Perpendicular AE is 10.2; which multiplied together, the Product is 198.9, that is, 198 ſuperficial Feet, and 9 tenth Parts, the Content.

DEMONSTRATION.

If DC be extended to F, making CF equal to DE, and a Line drawn from B to F; ſo will the Triangle CBF be equal to the Triangle ADE, and the Parallelogram AEFB be equal to the Rhombodies ABCD. Which was to be proved.

---

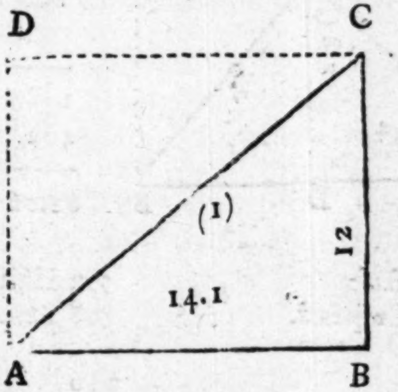
§ V. *Of a* TRIANGLE.

A Triangle is a Figure having three Sides and three Angles. Triangles are right-angled or oblique angled. Right-angled Triangles are ſuch as have one right-Angle. Oblique-angled Triangles are ſuch as have their Angles either acute or obtuſe. An obtuſe Angle is greater than a right Angle, that is, it is more than 90 Degrees; and an acute Angle is leſs than a right Angle. To find the ſuperficial Content thereof, this is

The

The RULE.

Let the Triangle be of what kind ſoever, multiply the Baſe by half the Perpendicular, or half the Baſe by the whole Perpendicular; or multiply the whole Baſe by the whole Perpendicular, and take half the Product: Any of theſe three Ways will give the Content.

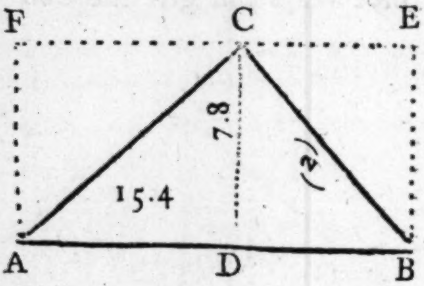


Let ABC be a right-angled Triangle whoſe Baſe is 14.1 Feet, and the Perpendicular 12 Feet, multiply 14.1 by 6, half the Perpendicular, and the Product is 84.6 Feet, the Content. Or multiply 14.1 by 12, the Product is 169.2; the half thereof is 84.6, the ſame as before.

14.1 Baſe.	14.1 Baſe.
6 Half Perpendicular.	12 Perpendicular.
84.6 Product.	169.2 Product.
	84.6 Half.

By Scale *and* Compaſſes.

Extend the Compaſſes from 2 to 14.1, that Extent will reach the ſame Way from 12 to 84.6 Feet, the Content.



15.4 Baſe,
3.9 half Perpend.

1386
462

60.06 Product.

15.4 Baſe
7.8 Perpend.

1232
1078

120.12

60.06 Half.

7.7 Half Baſe.
7.8 Perpend.

616
539

60.06 Product.

Let ABC (*Fig.* 2.) be an oblique-angled Triangle given, whoſe Baſe is 15.4, and the Perpendicular 7.8 ; if 15.4 be multiplied by 3.9, (half the Perpendicular) the Product will be 60.06 for the Area or ſuperficial Content : Or, if the Perpendicular 7.8 be multiplied into half the Baſe 7.7, the Product will be 60.06, as before : Or, if 15.4, the Baſe, be multiplied by the whole Perpendicular 7.8, the Product will be 120.12, which is the double Area ; the Half thereof is 60.06 Feet, as before.

See the WORK.

By Scale *and* Compaſſes.

Extend the Compaſſes from 2 to 15.4, that Extent will reach from 7.8 to 60 Feet, the Content.

DEMONSTRATION.

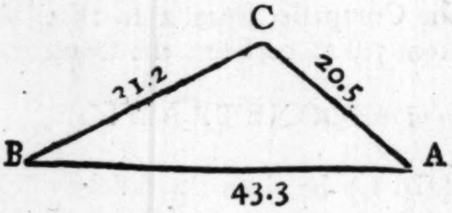
If AD (*Fig.* 1.) be drawn parallel to BC, and DC parallel to AB; the Triangle ADC ſhall be equal to the given Triangle ABC. Hence the Parallelogram ABCD is double to the given Triangle; therefore half the Area of the Parallelogram is the Area of the Triangle. In *Fig.* 2. the Parallelogram ABEF is alſo double to the Triangle ABC; for the Triangle ACF is equal to the Triangle ACD, and the Triangle BCE is equal to the Triangle BCD; therefore the Area of the Parallelogram is double to the Area of the given Triangle. Which was to be proved.

To find the Area of any plain Triangle, by having the three Sides given, without the Help of a Perpendicular.

RULE

Add the three Sides together, and take half that Sum; then ſubtract each Side ſeverally from that half Sum: Which done, multiply that half Sum and the three Differences continually, and out of the laſt Product extract the ſquare Root, which ſquare Root ſhall be the Area of the Triangle ſought.

Example. Let ABC be a Triangle, whoſe three Sides are as followeth, *viz.* AB 43.3, AC 20.5, and BC 31.2; the Area is required.



Sides	Differences
43.3	4.2
31.2	16.3
20.5	27.0

Sum 95.0

Half 47.5

Area 296.31

47.5 The half Sum.
27 Difference.

3325
950

1282.5 Product.
16.3 Difference.

38475
76950
12825

20904.75 Product.
4.2 Difference.

4180950
8361900

. . . . . .
87799.9500 (296.31
4

49) 477
441

586) 3699
3516

5923) 18395
17769

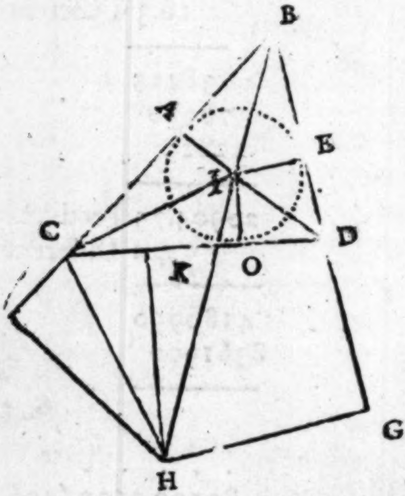
59261) 62600
59261

3339 Remains.

DEMON-

DEMONSTRATION.

In the Triangle BCD, I ſay, if from the half Sum of the Sides you ſubtract each particular Side, and multiply the half Sum and the three Differences together continually, the ſquare Root of the Product ſhall be the Area of the Triangle.



Firſt, by the Lines BI, CI and DI, biſect the three Angles, which Lines will all meet in the Point I; by which Lines the given Triangle is divided into three new Triangles CBI, DCI, and BDI; the Perpendiculars of which new Triangles are the Lines AI, EI and OI, being all equal to one another, becauſe the Point I is the Center of the inſcribed Circle (by *Euclid. Lib.* 4. *Prop.* 4)Wherefore to the Side BC join CF equal to DE or DO; ſo ſhall BF be equal to half the Sum of the Sides, *viz.* $= \frac{1}{2}BC + \frac{1}{2}BD + \frac{1}{2}CD$.

And $BA=BF-CD$; for $CA=CO$ and $OD=CF$, therefore $CD=AF$, and $AC=BF-BD$; for $BE,=BA$, and $ED=CF$; therefore $BD=BA+CF$, and $CF=BF-BC$.

Then make CK=CE, and draw the Perpendiculars FH, GH and KH, and extend BI to H; becauſe the Angles FCK + FHK are equal to two right Angles, (for the Angles F and K are right Angles) equal alſo to FCK+ACO (by *Euclid*, I. 13.) And the Angles ACO+AIO are equal to two right Angles; therefore the Quadrangles FCKH and AIOC are alike; and the Triangles CFH and AIC are alſo ſimilar. And the Triangles BAI and BFH are likewiſe ſimilar.

From this Explanation, I ſay, the Square of the Area of the given Triangle will be BF *q* × IA *q*=BF × BA × CA × CF. In Words:

The Square of BF (the half Sum of the Sides) multiplied into the Square of IA (=IF=IO) will be equal to the half Sum multiplied into all the three Differences,

For IA : BA : : FH : BF, and IA : CF : : AC : FH; becauſe the Triangles are ſimilar. By *Euclid*, *Lib.* 6. *Prop.* 4.

Wherefore multiplying the Extreams and Means in both it will be IA *q* × BF × FH=BA × CA × CF × FH; but FH being on both Sides of the Equation, it may be rejected; and then multiply each Part by BF, it will be BF *q* × IA *q*=BF × BA × CA × CF. Which was to be demonſtrated.

---

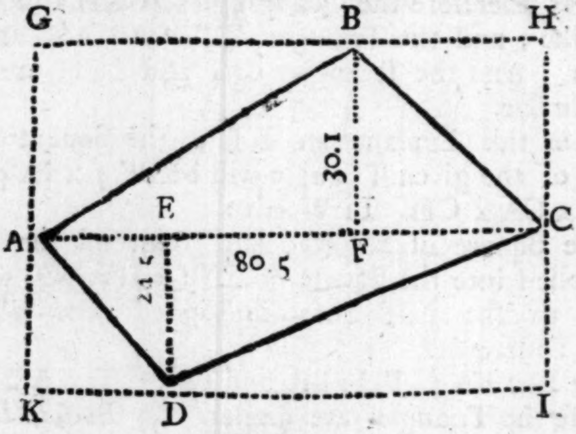
§ VI. *Of a* TRAPEZIUM.

A Trapezium is a Figure having four unequal Sides and oblique Angles. To find the Area or ſuperficial Content thereof, this is

The RULE.

Add the two Perpendiculars together, and take half the Sum, and multiply that half Sum by the Diagonal, or multiply the whole Sum by half the Diagonal, the Product is the Area. Or you may find the Areas of the two Triangles ABC and ACD, (by Section V.) and add thoſe two Areas together, the Sum ſhall be the Area of the Trapezium.

Or, *laſtly*, multiply the whole Diagonal by the Sum of the Perpendiculars, and take half the Product for the Area of the Trapezium.



BF=30.1
DE=24.5

Sum 54.6

Half 27.3
AC=80.5

1365
2184

Area 2197.65

Let ABCD be a Trapezium given, the Diagonal whereof is 80.5 and the Perpendicular BF 30.1, and the Perpendicular DE 24.5; theſe two added together the Sum is 54.6, the half thereof is 27.3, which multiplied by the Diagonal 80.5, the Product is 2197.65, which is the Area of the Trapezium; or if 4025, half the Diagonal, be multiplied by 54.6, the whole Sum of the Perpendiculars, the Product is 2197.65, the ſame as before.

By

By Scale *and* Compaſſes.

Extend the Compaſſes from 2 to 54.6; that Extent will reach from 80.5 to 2197.65, the Area.

DEMONSTRATION.

This Figure, ABCD, is compoſed of two Triangles, the Triangle ABC, is half the Parallelogram AGHC: Alſo the Triangle ACD is equal to half the Parallelogram ACIK, as was proved § V. Wherefore the Trapezium ABCD is equal to half the Parallelogram GHIK. To find the Area $HI = BF + DE$, therefore $\frac{1}{2} HI \times AC$ $(= KI = GH)$ = Area of the Trapezium. Which was to be proved.

---

§ VII. *Of* IRREGULAR FIGURES.

IRREGULAR Figures are all ſuch as have more Sides than four, and the Sides and Angles unequal. All ſuch Figures may be divided into as many Triangles as there are Sides, wanting two. To find the Area of ſuch Figures, they muſt be divided into Trapeziums and Triangles, by Lines drawn from one Angle to another, and ſo find the Areas of the Trapeziums and Triangles ſeverally, and then add all the Areas together, ſo will you have the Area of the whole Figure.

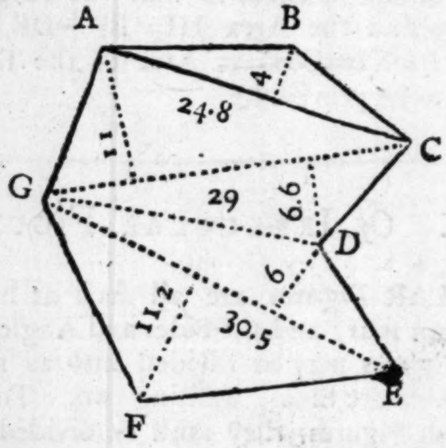
Let ABCDEFG be an irregular Figure given to be meaſured; firſt, draw the Lines AC and GD, and thereby divide the given Figure into two Trapeziums, ACGD and GDEF, and the Triangle ABC; of all which I find the Areas ſeverally.

Firſt, I multiply the Baſe AC by half the Perpendicular, and the Product is 49.6, the Area of the Triangle ABC.

Then, for the Trapezium, ACGD, the two Perpendiculars 11 and 6.6 added together make 17.6; the half thereof is 8.8, which multiplied by 29, the Diagonal, the Product is 255.2, the Area of that Trapezium.

And for the Trapezium GDEF, the two Perpendiculars, 11.2 and 6, added together make 17.2, the half thereof is 8.6, which multiplied by 30.5, the Diagonal, the Product is 262.3, the Area thereof. All theſe Areas added together make 567.1, and ſo much is the Area of the whole irregular Figure.

See the WORK.



24.8 Baſe AC.
2 half Perpendic.

49.6 Area of ABC.

11 } Perpendiculars
6.6 }

17.6 Sum.

8.8 Half
29 Diag. CG.

792
176

255.2 Area of ACGD.

11.2

11.2 } Perpendiculars.
6. }

17.2 Sum.
8.6 Half Sum.

30.5
8.6

1830
2440
262.30 Area of GDEF.
255.2 Area of ACGD.
49.6 Area of ABC.

567.1 Sum of the Areas.

The Figure being compoſed of Triangles and Trapeziums, and thoſe Figures being ſufficiently demonſtrated in the fifth and ſixth Sections aforegoing, it will be needleſs to mention any Thing of the Demonſtration thereof in this Place.

§ VIII. *Of* Regular Polygons.

Regular Polygons are all ſuch Figures as have more than four Sides, all the Sides and Angles thereof being equal Polygons, are denominated from the Number of their Sides and Angles.

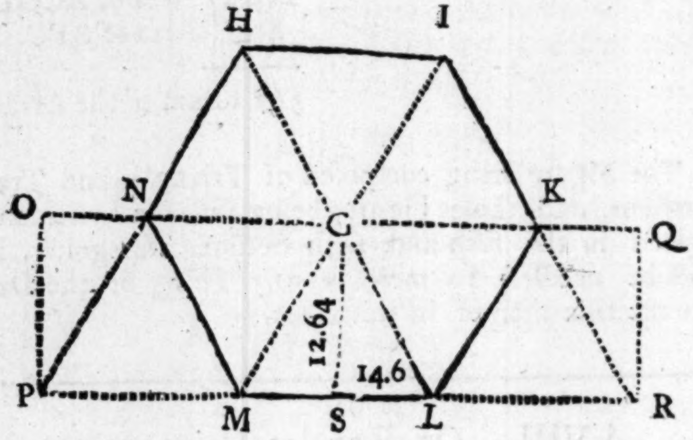
If the Figure conſiſts of	Equal Sides and Angles it is called a Regular
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Enneagon
10	Decagon
11	Endecagon
12	Dodecagon

To find the Area or ſuperficial Content of any Regular Polygon, this is

The

The RULE.

Multiply the whole Perimeter, or Sum of the Sides, by half the Perpendicular, let fall from the Center to the Middle of one of the Sides; or multiply the half Perimeter by the whole Perpendicular, and the Product is the Area.



14.6
3

43.8 Half Sum of the Sides.

12.64 The Perpend.
43.8 Half Sum.

10112
3792
5056

553.632 Area

14.6
6

87.6 Sum of the Sides.
6.32 Perpend.

1752
2628
5256

553.632 Area.

Let HIKLMN be a regular Hexagon, each Side thereof being 14.6, the Sum of all the Sides is 87.6, the half Sum thereof is 43.8, which multiplied by the Perpendicular GS 12.64, the Product is 553.632: Or if 87.6, the whole Sum of the Sides, be multiplied by half the Perpendicular 6.32, the Product is 553.632: the ſame as before; which is the Area of the given Hexagon.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 12.6, that Extent will reach from 43.8, the ſame Way to 553.63: Or extend from 2 to 12.6, that Extent will reach from 87.6 to 553.63.

DEMONSTRATION.

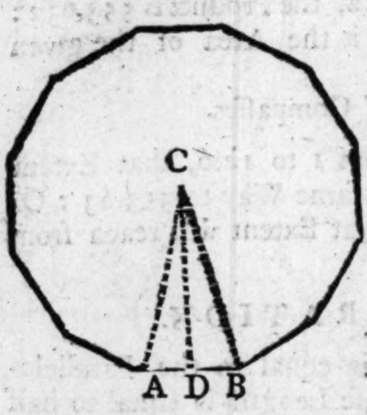
Every regular Polygon is equal to the Parallelogram, or long Square, whoſe Length is equal to half the Sum of the Sides, and Breadth equal to the Perpendicular of the Polygon, as appears by the foregoing Figure; for the Hexagon HIKLMN is made up of ſix equilateral Triangles: And the Parallelogram OPQR is alſo compoſed of ſix equal and equilateral Triangles, that is, five whole ones, and two half ones; therefore the Parallelogram is equal to the Hexagon.

A TABLE *for the more ready finding the Area of a Polygon.*

Number of Sides.	Names.	Multiplier.
3	Trigon	.433013
4	Tetragon	1.000000
5	Pentagon	1.720477
6	Hexagon	2.598076
7	Heptagon	3 633931
8	Octagon	4.828424
9	Enneagon	6.181821
10	Decagon	7.694205
11	Endecagon	9.365653
12	Dodecagon	11.196150

Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

How to find theſe tabular Numbers.



Theſe Numbers are found by Trigonometry, thus: Find the Angle at the Center of the Polygon, by dividing 360 Degrees by the Number of Sides of the Polygon.

Example. Suppoſe each Side of the Dodecagon annexed be 1, and the Area be required.

Divide 360 by 12, (the Number of Sides) the Quotient is 30 Degrees for the Angle ACB; the half thereof is 15, the Angle DCB, whoſe Complement to 90 Degree is 75 Degrees, the Angle CBD: Then ſay,

As s. DCB 15 Degrees, - - - C. Ar. 0.587004
To .5, the half Side DB Log. - - - 1.698970
So is s. CBD 75 Degrees - - - - - - 9.984944

To the Perpendicular CD 1.866025 0.270918

Then 1.866025 multiplied by .5, (the half Side) the Product is .9330125, =the Area of the Triangle ACB, which again multiplied by 12, the Number of Sides in the Dodecagon, produces 11.196150 * the Area of the Dodecagon required whoſe Side is = 1.

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* In all the former Editions, there was a great Error ſuffered to paſs here.

§ IX. *Of a* CIRCLE.

A Circle is a plain Figure, contained under one Line, which is called a Circumference, unto which all Lines, drawn from a Point in the Middle of the Figure, called the Center, and falling upon the Circumference thereof, are all equal the one to the other. The Circle contains more Space than any plain Figure of equal Compaſs.

Problem 1. Having the Diameter and Circumference, to find the Area.

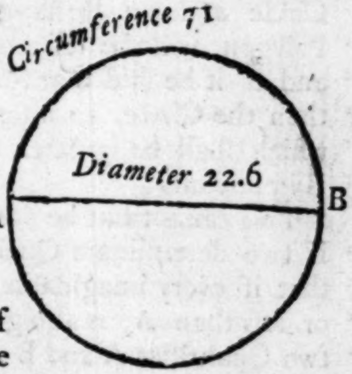
The RULE.

Every Circle is equal to a Parallelogram, whoſe Length is equal to half the Circumference, and the Breadth equal to half the Diameter; therefore multiply half the Circumference by half the Diameter, and the Product is the Area of the Circle.

35.5 Half Circumference
11.3 Half Diameter.

1065
355
355

401.15 Area.



Thus, if the Diameter of a Circle (that is, the Line drawn croſs the Circle thro' the Center) be 22.6, and if the Circumference be 71, the half of 71 is 35.5, and the half of 22.6 is 1.3; which multiplied together, the Product is 401.15, which is the Area of the Circle.

DEMONSTRATION.

Every Circle may be conceived to be a Polygon of an infinite Number of Sides, and the Semidiameter muſt be equal to the Perpendicular of ſuch a Polygon, and the Circumference of the Circle equal to the Periphery of the Polygon; therefore half the Circumference, multiplied by half the Diameter, gives the Area, as aforeſaid.

Or, with *F. Ignat. Gaſton. Pardies*) “ Every Circle “ is equal to a Rectangled-Triangle, one of whoſe Legs “ is the Radius, and the other a right Line equal to “ the Circumference of the Circle: For ſuch a Triangle will be greater than any Polygon inſcribed, “ and leſs than any Polygon circumſcribed, by the “ 24th, 25th, 26th, and 27th Articles of the fourth “ Book of his Elements of Geometry) and therefore “ muſt be equal to the Circle.

“ For (ſays he) ſhould it be greater than the Circle, “ be the Exceſs as little as it will, a Polygon may be “ circumſcribed, whoſe Difference, from the Circle, “ ſhall be yet leſs than the Difference between that “ Circle and the Rectangle-Triangle; and that that “ Polygon will be leſs than the Triangle is abſurd; “ and if it be ſaid that this rectangled Triangle is leſs “ than the Circle, an inſcribed Polygon may be made “ which ſhall be greater than that Triangle; which “ is impoſſible.

“ This cannot but be admitted as a Principle, That “ if two determinate Quantities, A and B, are ſuch, “ that if every imaginable Quantity, which is greater “ or leſs than A, is alſo greater or leſs than B, theſe “ two Quantities A and B muſt be equal.

“ And this Principle being granted, which is in a “ manner ſelf-evident, it may directly be proved, “ that the Triangle (before mentioned) is equal to the “ Circle; becauſe every imaginable inſcribed Figure, “ which is leſs than the Circle, is alſo leſs than the “ Triangle; and every circumſcribed Figure greater “ than the Circle, is alſo greater than the Triangle.”

Problem 2. Having the Diameter of a Circle, to find the Circumference.

As 7 to 22, ſo is the Diameter to the Circumference.

Or, as 113 to 355, ſo is the Diameter to the Circumference.

Or, as 1 to 3.141593, ſo is the Diameter to the Circumference.

Let the Diameter (as in the former Circle) be 22.6, this multiplied by 22, and the Product is 497.2; which divided by 7, gives 71.028 for the Circumference, Or (by the ſecond Proportion) if 22.6 be multiplied by 355, the Product will be 8023; this divided by 11.3, the Quotient is 71, the Circumference. Or (by the third Proportion) if 22.6 be multiplied into 3.141593, the Product is 71.0000018, the Circumference; which two laſt Proportions are the moſt exact.

22.6
22
———
452
452
———
7) 497.2 (71.028

3.141593
22.6
———
18849558
6283186
6283186
———
71.0000018

355
22.6
———
2130
710
710
———
113) 8023.01 (71
791
———
113
113
———
. . .

By Scale *and* Compaſſes.

Extend the Compaſſes from 7 to 22, or from 113 to 355, or from 1 to 3.14159; that Extent will reach from 22.6 to 71.

The Proportion of the Diameter of a Circle, to the Circumference, was never yet exactly found, notwithſtanding many eminent learned Men have laboured very far therein; amongſt which the excellent *Van Culen* hath hitherto outdone all, in his having calculated the ſaid Proportion to 36 Places of Decimals, which are engraven upon his Tomb-ſtone in St. *Peter*'s Church in *Leyden*; which Numbers are theſe,

Diameter.

1.00000.00000.00000.00000.00000,00000.00000

Circumference.

3.14159.26535.89793.23846.26433.83279.50288.

Of which large Number theſe ſix Places, 3.14159, anſwering to the Diameter 1.00000, may be ſufficient; of the three Proportions, as 7 to 22.113 to 355, and 1 to 3.14159, I ſhall leave my Reader to uſe which of them he pleaſes, but ſhall commend the laſt two as moſt exact, tho' the firſt be moſt in Uſe; but in the following Work I ſhall ſometimes uſe one of them, and ſometimes another, but for the moſt Part that of *Van Culen*, as being moſt exact.

Problem 3. Having the Circumference of a Circle to find the Diameter.

As 1 is to .318309, ſo is the Circumference to the Diameter.

Or, as 355 to 113, ſo is the Circumference to the Diameter.

Or, as 22 to 7; ſo is the Circumference to the Diameter.

Let the Circumference be 71, (as in the former Circle) if .318309 be multiplied by 71 (as by the firſt Proportion) the Product will be 22.599939 for the Diameter. Or, by the ſecond Proportion, 113 multiplied by 71, the Product is 8023; which divided by 355, the Quotient will be 22.6 the Diameter. Or, by the third Proportion, 71 multiplied by 7, the Product is 497; this divided by 22, the Quotient is 22.5909, the Diameter.

.318309

.318309	113	71
71	71	7
318309	113	22)497(22.59
2228163	791	44
22.599939	355)8023(22.6	57
	710	44
	923	130
	710	110
	2130	200
	2130	198
		2

Thus, by both the firſt Proportions, the Diameter is 22.6; but by the laſt it falls ſomething ſhort.

By Scale *and* Compaſſes.

Extend the Compaſſes from 3.14159 to 1, that Extent will reach from 71 to 22.6, which is the Diameter ſought.

Or you may extend from 1 to .318309.

Or from 22 to 7.

Or from 355 to 113; the ſame will reach from 71 to 22.6 as before.

NOTE, That if the Circumference be 1, the Diameter will be .318309.

Problem 4. Having the Diameter of a Circle, to find the Area.

All Circles are in Proportion one to another, as are the Squares of their Diameters, (by *Euclid*, 12. 2.) Now the Area of a Circle, whoſe Diameter is 1, will be .785398, according to *Van Culen*'s Proportion, before mentioned; but for Practice .7854 will be ſufficient: Therefore,

As 1 (the Square of the Diameter 1) is to .7854, ſo is 510.76 (the Square of 22.6, the Diameter of the given Circle) to 401.51. (the Area of the given Circle) But,

According to *Metius*'s Proportion;

As 452 : 355 :: 510.76 : 401.15, the ſame as before.

But, if you uſe *Archimedes*'s Proportion.

As 14 : 11 : : 510.76 : 401.31 ; which Area is greater than by the two former Proportions; tho' in ſmall Circles this is near enough the Truth.

See the Working of all theſe.

22.6 Diameter of the former Circle.
22.6
———
1356
452
452
———
510.76 The Square of the ſaid Diameter.

As 1 : 7854 : : 510.76
.7854
———
204304
255380
408608
357532
———
401.150904 The Area.

By Scale *and* Compaſſes,

The Extent from 1 to 22.6, being twice turned over from .7854, will fall at the laſt upon 401.15, the Area.

113
4
452

As 452 : 355 : : 510.76
355
255380
255380
153228
452) 181319.80 (401.15
1808
519
452
678
452
2260
2260
. . . .

As 14 : 11 : : 510.76
11
14) 561836 (401.31
56
18
14
43
42
16
14
2

Problem 5. Having the Circumference of a Circle, to find the Area.

Becauſe the Diameters of Circles are proportional to the Circumferences; that is, as the Diameter of one Circle is to its Circumference, ſo is the Diameter of another Circle to its Circumference. Therefore the Areas of Circles are to one another as the Squares of the Circumferences. And if the Circumference of a Circle be 1, the Area of that Circle will be .07958; then the Square of 1 is 1. and the Square of 71 (the Circumference of the former Circle) is 5041. Therefore it will be,

```
          Sq. Cir.   Area Sq. Circumf. Square
          As     1 : .07958 : : 5041
                            5041
                          -------
                            7958
 22                        31832
  4                       397900
 ---                      ---------
 88                       401.16278 Area.
```

Or thus:

```
        As 88 : 7 : : 5041
                         7
                    ----------
                  88|35287|400.98 Area.
                     352
                    -----
                       870
                       792
                       ---
 355                    780
   4                    704
 ----                   ---
 1420                    76
                            Area.
```

Or, As 1420 : 113 : : 5041 : 401.15.

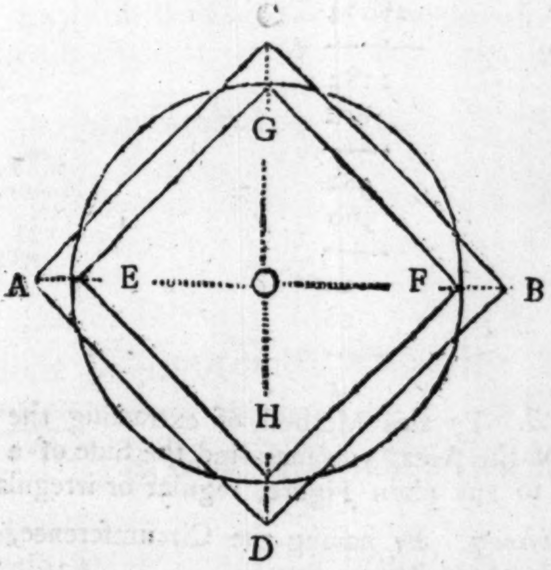
Problem 6. By having the Diameter, to find the Side of a Square that is equal in Area to that Circle.

If the Diameter of a Circle be 1, the Side of a Square equal thereunto will be .8862 : Therefore,

As 1 : .8862 : : 22.6 (the Diameter.)
22.6

53172
17724
17724

To 20.02812 The Side of the Square AC.



Let the Diameter of the Circle EF or GH be 22.6 (as before) to find the Side of the Square AC, AD, *&c.* If .8862 be multiplied by 22.6, the Product is 20.02812, which is the Side of a Square, equal in Area to the Circle given; for if 20.02812 be multiplied ſquare-wiſe, that is, by itſelf, it will produce 401.1255907344, which is nearly equal to the Area found in the laſt *Problem.*

You may find the Side of the Square equal, by extracting the ſquare Root out of the Area of the given Circle.

```
          .  .  .
          401.15(20.0287295 Side of the Square.
          4
          -------
   4002)01.1500
            8004
          --------
    40048)349600
    ....  349600
          -------
           29216
           28034
          -------
            1182
             801
            ----
             381
             360
            ----
              21
              20
            ----
               1
```

N. B. By this Method of extracting the ſquare Root of the Area, you may find the Side of a Square equal to any plain Figure, regular or irregular.

Problem 7. By having the Circumference, to find the Side of the Square equal.

If the Circumference of a Circle be 1, the Side of the Square equal will be .2821. Therefore,

```
As 1 : .2821 : : 71 (the Circumference.)
                 71
           ----------
               2821
             19747
          -----------
             20.0291 The Side of the Square.
```

Problem 8. Having the Diameter to find the Side of a Square which may be inſcribed in that Circle.

If the Diameter of a Circle be 1, the Side of the Square inſcribed will be .7071. Therefore,

As 1 : 7071 : : 22.6
22.6
42426
14142
14142
To 15.98046 the Side EG inſcribed.

Or, if you ſquare the Semidiameter, and double that Square, the Square Root of the doubled Square will be the Side of the Square inſcribed. For (by *Euclid*, 1.47.) the Square of the Hypothenuſe EG is equal to the Sum of the other two Legs, EO and OG.

11.3 Semidiameter.
11.3
339
113
113
127.69 The Square of EO, which double, becauſe
2 [EO=GO :
255.38(15.98 Root, which is the Side of the Squ.
1
25) 155
125
309) 3038
2781
3188) 25700
25504
196

Problem 9. Having the Circumference to find the Side of a Square which may be inſcribed.

If the Circumference be 1, the Side of the Square inſcribed will be .2251. Therefore,

As 1 : .2251 :: 71
71
———
2251
15757
———
15 9821 The Side of the Square EG.

Becauſe that in each of the four laſt *Problems*, *viz.* the 6th, 7th, 8th, and 9th, there is a Proportion laid down, it will be eaſy to work them with Scale and Compaſſes, for if you extend the Compaſſes from the firſt to the ſecond, that Extent will reach from the third to the fourth. As in the laſt *Problem*, where the Proportion is as 1 to .2251, ſo is 71 to the Side of the Square 15.9821. Here extend the Compaſſes from 1 to .2251; that Extent will reach from 71 to 15.98; and ſo of the reſt. But the fifth muſt be wrought like the fourth, thus; extend the Compaſſes from 1 to 71, that Extent turned over the ſame Way from .07958, will fall at the laſt upon 401.15.

Problem 10. Having the Area, to find the Diameter.

If the Area of a Circle be 1, the Square of the Diameter thereof is 1.2732. Therefore,

```
Area. Sq. Diam. Area.      . :  . . .
As 1 : 1.2732 : : 401.15   510.744180(22.599 the Diam.
           401.15          4
          -------          -----
          63660        42) 110
        12732               84
       12732               -----
      50928           445) 2674
    -----------            2225
    510.744180             ------
                      4509( 44941
                             40581
                     45189) 436080
                             406701
                             -------
                             .29379
```

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 1.2732; that Extent will reach from 401 15 to 510.74, *&c.* Then divide the Space between 1 and 510.74 into two equal Parts, and you'll find the middle Point at 22.6. Or you may divide the Space upon the Line of Numbers, between 401.55 and .7854, into two equal Parts, and one of thoſe Parts will reach from 1 to 22.6, the Diameter ſought.

Problem 11. Having the Area, to find the Circumference.

If the Area of a Circle be 1, the Square of the Circumference will be 12.56637. Therefore,

```
Ar. Sq. Circum.  Area.     . .  . . . . Circumf.
As 1 : 12.56637 : : 401.15   5040.99932550(70.9999 Root
            401.15           49
         ---------           ------
          6283185      1409) 14099
        1256637               12681
       1256637               -------
      50265480        14189) 141893
     ------------             127701
     5040.99932550            --------
                     141989) 1419225
                              1277901
                             ---------
                    1419989) 14132450
                              12779901
                             ---------
                              1352549
```

By

By Scale *and* Compaſſes.

Divide the Space between 401.15 and 0.7958, upon the Line, into two equal Parts; one of thoſe Parts will reach from 1 to 71, the Circumference ſought.

Problem 12. Having the Area, to find the Side of a Square inſcribed,

If the Area of a Circle be 1, the Area of a Square inſcribed within that Circle will be .6366. Thereſore,

```
As 1 : 401.15 : : .6366
            .6366
           240690
          240690
         120345
        240690
        • . . . . .
        255.372090 (15.98 Root, which is the Side of the
        1                              [Square ſought.
     25)155
        125
    309)3037
        2781
   3188)25620
        25504
        11690
```

The ſame Reaſon may be given for the laſt Proportion that was given before for the Proportion of Circles to the Squares of their Diameters and Circumferences; for not only the Squares of the Diameters and Circumferences are in Proportion to the Circles they belong to, but alſo all Figures inſcribed or circumſcribed have the Squares of their like Sides proportional to the Circles they are inſcribed in, or circumſcribed about, and alſo to the Figures themſelves: The Square of any Side of one Figure is to the Area of that Figure, as the Square of the like Side of another ſimilar Figure is to the Area thereof, as you may find proved at large in *Euclid, Sturmius's Matheſis Enucleata*, and other Authors, but will be too large to inſert in this Place.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 401.15, that Extent will reach from .6366 to 255.37; the half Space between that and 1 is at 15.98, the Side of the Square.

Problem 13. Having the Side of a Square, to find the Diameter of the circumſcribing Circle.

If the Side of a Square be 1, the Diameter of a Circle that will circumſcribe that Square will be 1.4142, Therefore,

$$\begin{array}{r}
\text{As } 1 : 1.4142 : : 15.98 \\
15.98 \\
\hline
113136 \\
127278 \\
70710 \\
14142 \\
\hline
22.598916
\end{array}$$

22.598916 The Diameter ſought.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 1.4142, and that Extent will reach from 15.98 to 22.6, the Diameter ſought.

Problem 14. Having the Side of a Square, to find the Diameter of a Circle equal.

If the Side of a Square be 1, the Diameter of a Circle equal thereunto will be 1.128. Therefore,

$$\begin{array}{r}
\text{Side. Diam. Side of a Square.} \\
\text{As } 1 : 1.128 : : 20.0291 \\
1.128 \\
\hline
1602328 \\
400582 \\
200291 \\
200291 \\
\hline
22.5928248
\end{array}$$

22.5928248 Diameter,

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 1.128; that Extent will reach from 20.0291 (the Side of the Square given) to 22.6, the Diameter of the Circle ſought.

Problem 15. Having the Side of a Square, to find the Circumference of the circumſcribing Circle.

If the Side of a Square be 1, the Circumference of a Circle that will encompaſs that Square will be 4.443. Therefore,

Side Sq. Circum. Side Sq.
As 1 : 4.443 : : 15.98
15.98

35544
39987
22215
4443

70.99914 The Circum.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 4.443, that Extent will reach from 15.98 to 71, the Circumference.

Problem 16. Having the Side of a Square, to find the Circumference of a Circle that will be equal thereunto.

If the Side of the Square be 1, the Circumference of a Circle that will be equal thereunto ſhall be 3.545. Then,

As 1 : 3.545 : : 20.0291
3.545

1001455
801164
1001455
600873

71.0031595 The Circum.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 3.545, that Extent will reach from 20.0291 to 71, the Circumference ſought.

In ſeveral of the foregoing *Problems*, where the Diameter and Circumference is required, the Anſwers are not exactly the ſame as the Diameter and Circumference of the given Circle, but are ſometimes too much and ſometimes too little, as in the two laſt Problems, where the Anſwers in each ſhould be 71, the one being too much, and the other too little. The Reaſon of this is, the ſmall Defect that happens to be in the Decimal Fractions, they being ſometimes too great, and ſometimes too little; yet the Defect is ſo ſmall, that it is needleſs to calculate them to more Exactneſs.

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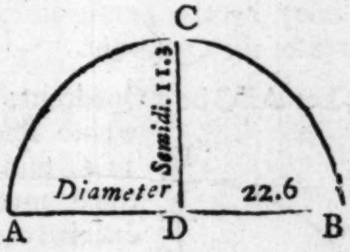
§ X. *Of a* SEMICIRCLE.

TO find the Area of a Semicircle, this is

The RULE.

Multiply the fourth Part of the Circumference of the whole Circle (that is, half the Arch-line) by the Semidiameter, the Product is the Area.

Let ABC be a Semicircle whoſe Diameter is 22.6, and the half Circumference, or Arch-line ACB, is 35.5, the half thereof is 17.75, which multiply by the Semidiameter 11.3, and the Product is 200.575, the Area of the Semicircle.



17.75 The half Arch-line.
11.3 The Semidiameter.

5325
1775
1775

200.575 The Area of the Semicircle.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 11.3; that Extent will reach from 17.75 to 200.575. the Area.

If only the Diameter of the Semicircle be given, you may ſay by the *Rule of Three*.

As 1 is to .3927, ſo is the Square of the Diameter to the Area.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to the Diameter 22.6; that Extent turned twice from .3927, will reach at the laſt to 200.575.

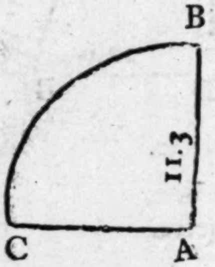
§ XI. *Of a* QUADRANT.

TO find the Area of a Quadrant, or fourth Part of a Circle, this is

The RULE.

Multiply half the Arch-line of the Quadrant, (that is, the eighth Part of the Circumference of the whole Circle) by the Semidiameter, and the Product is the Area of the Quadrant.

Let ABC be a Quadrant, or fourth Part of a Circle, whoſe Radius, or Semidiameter is 11.3, and the half Arch-line 8.875; theſe multiplied together, the Product is 100.2875 for the Area.



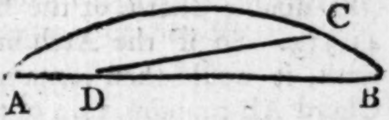
Theſe are the Rules and Ways commonly given for finding the Area of a Semicircle and Quadrant; but, I think, it is as good a Way to find the Area of the whole Circle, and then take half that Area for the Semicircle, and fourth Part for the Quadrant.

Before I proceed to ſhew how to find the Area of the Sector, and Segment of a Circle, I ſhall ſhew how to find the Length of the Arch-line both *Geometrically* and *Arithmetically*.

To

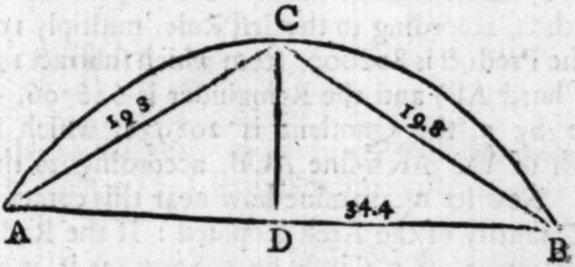
To find the Length of the Arch-line *Geometrically.*

Divide the Chord-line AB into four equal Parts, and ſet one of theſe Parts from B to C, and draw a Line from C to three of thoſe Parts at D; ſo ſhall CD be equal to half the Arch-line ACB.



To find the Length of the Arch-line *Arithmetically.*

Multiply the Chord of half the Segment AC or CB by 8, and from the Product ſubtract the Chord of the whole Segment AB, and divide the Remainder by 3, the Quotient is the Arch-line ACB ſought.



19.8 AC
8

158.4
34.4 AB

3) 124

Arch-line 41.333

Another Way.

From the double Chord of half the Segment's Arch, ſubtract the Chord of the Segment, one third Part of the Difference added to the double Chord of half the Segment's Arch, the Sum is the Arch-line of the whole Segment.

Thus. if AC 19.8 be doubled, it makes 39.6; from which if you ſubtract 34.4, the remainder is 5.2, which divided by 3, the Quotient is 1.733; this added to 39.6 (the double Chord of the half Segment) the Sum is 41.333. So if the Arch-line ACB was ſtretched out ſtrait, it would then contain 41.333 ſuch Parts as the Chord AB contains 34.4 of the like Parts.

Theſe two Rules may very eaſily be proved out of the Table of natural Sines: thus,

Suppoſe (in the former Figure) the Arch ACB to contain 120 Degrees, the natural Sine of half, *viz.* of 60 Degrees, is 86602, which being doubled is 173204, which is the Chord of the whole 120 Deg. that is AB. Then, to find the Chord of the half Arch, *viz.* AC 60 Degrees, the half of it 30 Deg. the natural Sign thereof is 50000, which doubled makes 100000; for the Chord AC; then, according to the firſt Rule, multiply 100000 by 8 the Product is 800000; from which ſubtract 173204 (the Chord AB) and the Remainder is 626796, which divide by 3, the Quotient is 208932, which is the Length of the Arch-line ACB, according to the firſt Rule. Now let us examine how near this comes to the true Quantity of the Arch propoſed: If the Radius or Semidiameter of a Circle be 100000 (as it is in the Table of Signs) then the Circumference will be 628318; and becauſe 120 Degrees is the third Part of the Circle, take the third Part of 628318, which is 2094.39, the true Quantity of the Arch ACB in ſuch Parts as the Semidiameter contains 100000, and differs from that before found 507. which is a Thing inconſiderable in *Practical Menſuration.* The latter of the foregoing Rules agrees exactly with the former, and therefore the Difference will be the ſame as above; either of the Rules gives the Quantity of the Arch-line too little; and the greater the Arch the greater the Error, If you know the Degrees that are contained in the Segment's Arch, and would have the Arch-line very exactly, you may reaſon thus by the *Rule of Three*;

As the Circle's Periphery in Degrees: is to its Periphery in equal Parts : : ſo is the Arch in Degrees and decimal Parts : to the ſame Arch in equal Parts.

Suppoſe the Circumference of a Circle be 71, and ſuppoſe the Arch to contain 52 Degrees, 15 Minutes, (the Decimal of 15 Minutes is .25) then ſay,

Deg. Parts. Deg.
As 360 : 71 : : 52.25
71

5225
36575

36|0) 370|9,75 (10.305 *fere.*
36

109
108

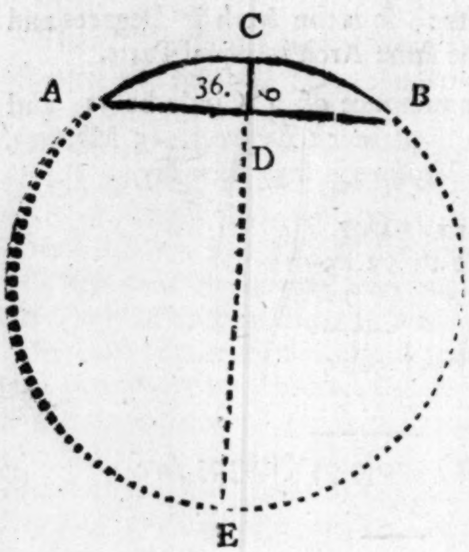
175

So the 52 Degrees, 15 Minutes, will contain 10.305 of ſuch Parts as the Circumference contains 71.

Thus have I ſhewed ſeveral Ways of finding the Meaſure of the Curve-line of any Part of a Circle very near the Truth. The next Thing I ſhall ſhew, is,

How to find the Diameter of a Circle, *by having the Chord and verſed Sine of the Segment, Arithmetically.*

Becauſe the Chord AB cuts the Diameter EC at right Angles, therefore the Semichord AD or DB is a mean proportional Line between the Parts of the Diameter CD and (DE, by *Euclid*, 6 13.) Therefore, if you ſquare the Semichord AD or DB; and divide that Square by the verſed Sine CD, the Quotient will be the Part of the Diameter wanting; to which add the given verſed Sine CD, and the Sum is the Diameter ſought.



Example.

Let ACB be a Segment given, whoſe Chord AB is 36, and the verſed SineCD 6; half 36 is 18, which ſquared, makes 324; this divided by 6, the Quotient is 54; to which add 6, the Sum is 60, the Diameter of the Circle CE.

18 Half the Chord.
18

144
18

6) 324 The Square of AD.

54 The Part wanting DE.
6 The verſed Sine CD add.

60 The Diameter CE.

---

§ XII. *Of a Sector of a* CIRCLE.

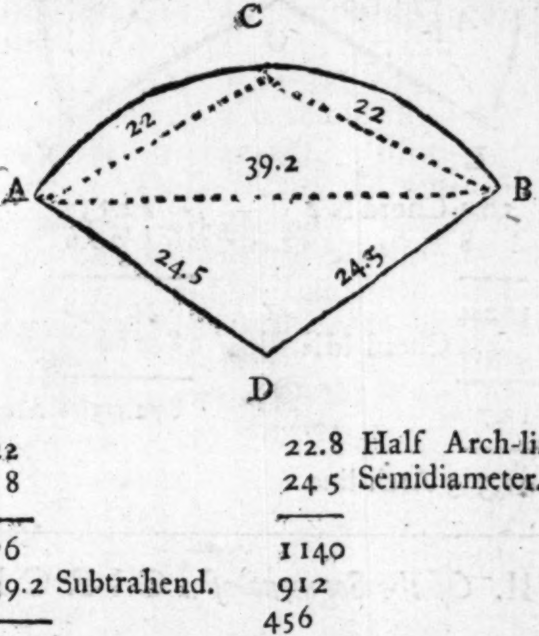
A Sector of a Circle is Comprehended under two Radii, or Semidiameters, which are ſuppoſed not to make one right Line, and a Part of the Circumference; whence a Sector may be either leſs or greater than a Semicircle. To find the Area or ſuperficial Content thereof, this is

The

RULE.

Multiply half the Arch-line by the Semidiameter, and the Product is the Area.

Let ADBC be the Sector of a Circle given, whoſe Semidiameter AD or BD is 24.5, and the Arch-line ACB (by the firſt Rule, *Page* 115.) I find to be 45.6, the Half thereof, 22.8, being multiplied by 24.5, (the Semidiameter) the Product is 558.6, which is the Area of the Sector ACBD.

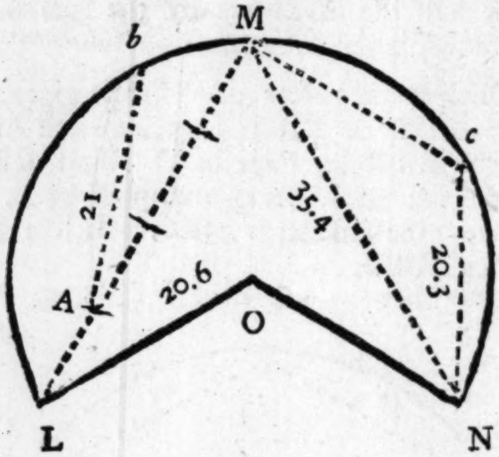


```
   22
    8
  ----
  176
   39.2 Subtrahend.
  -----
3) 136.8
  -----
   45.6 Arch-line.
```

```
  22.8 Half Arch-line.
  24 5 Semidiameter.
  ----
  1140
  912
 456
 -----
 558.60 The Area.
```

Again. Let LMNO be a Sector greater than a Semicircle whoſe Semidiameter LO or NO is 20.6, and Line *b a* equal to a fourth Part of the Arch-line L *b* MN 21, the double whereof is 42, equal to the Arch-line L *b* M or M *c* N, or by the Arithmetical Rule in *Page* 115, the ſaid Arch is found to be 42.333, which multiplied by 20.6, the Semidiameter makes 872.0598 for the Area of the Sector LMNO.

See the following Work.



20.3 Chord N *c*
8

162.4
35.4 Chord MN

3) 12.70

42.333 Arch-line.

42.333
20.6

253998
84666

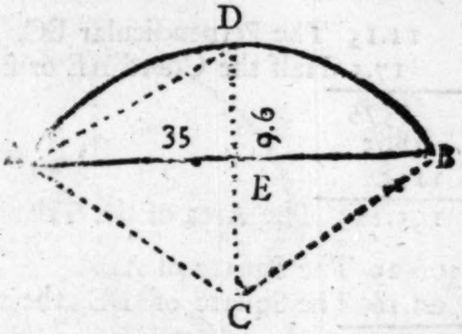
872.0598 Area.

§ XIII. *Of the Segment of a* CIRCLE.

A Segment of a Circle, is a Part terminated by a right Line leſs than the Diameter, called a Chord, and by a Part of the Circumference.

To find the Area of the Segment of a Circle, you muſt firſt find the Center of the whole Circle, and draw the two Semidiameters, thereby completing the Sector, as in the following Figure. Then (by the laſt Section) find the Area of the whole Sector CADBC, and then (by Sect. 5) find the Area of the Triangle ABC, and ſubtract the Area of the Triangle out of the Area of the Sector, the Remainder is the Area of the Segment.

Otherwiſe



Otherwiſe you may, without deſcribing the Figure, find the Semidiameter of the Circle by the Arithmetical Rule, *Page* 118, and by the Arithmetical Rule, *Page* 115, find the Arch-line ; then multiply half the Arch-line by the Semidiameter, ſo have you the Area of the Sector. Then ſubtract the verſed Sine from the Semidiameter, the Remainder is the Perpendicular of the Triangle ; and multiply the half Chord by the Perpendicular, the Product is the Area of the Triangle. Then ſubtract the Area of the Triangle from the Area of the Sector, and the Remainder is the Area of the Segment. See the WORK.

2) 35=AB.

17.5
17.5

875
1225
175

96) 306.25 (31.9
9.6 add

182
865 41.5 The Diameter of the Circle.

1 20.75 The Semidiameter.
9.6 DE Subtrahend.
11.15 Remains the Perpendic. EC.

11.15 The Perpendicular EC.
17.5 Half the Chord AE or EB.

5575
7805
1115

195.125 The Area of the Triangle.

306.25 The Square of AE.
92.16 The Square of DE, the verſed Sine.

398.41 Sum.

The Square Root thereof is 19.96 the Chord AD.
8

159.68
Subt. 35 the Chord AB.

3) 124.68

2) 41.56 the Arch-line.

20.78 Half.
20.75 Semidiameter.

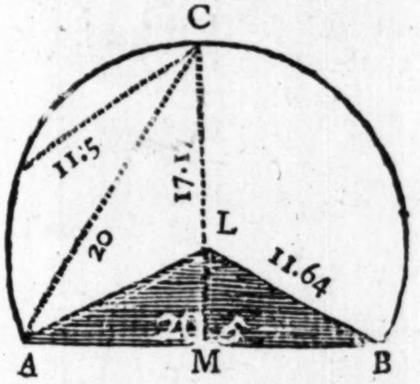
10390
14546
4156

From 431.1850 Area of the (Sector.

Subtract 195.125 Area of the Tri.

Remains 236.060 Area of the Seg.

Again, Let MACBM be a Segment greater than a Semicircle; obſerve the former Rules in all Reſpects, as in the laſt *Example*, only inſtead of Subtracting the Area of the Triangle out of the Area of the Sector, here you muſt add it thereunto, as may plainly appear by the following Figure.



11.5
8

92.0
20

3)72

24 Half Arch-line.

11.64 Semidiam.
24

4656
2328

17.17
11.64

5.53 LM.

Area of the Sector LACBL 279.36

10.25 Half the Baſe MA.
5.53 The Perpendicular LM.

3075
5125
5125

56.6825 The Area of the Triangle ALM.
279.36 The Area of the Sector add.

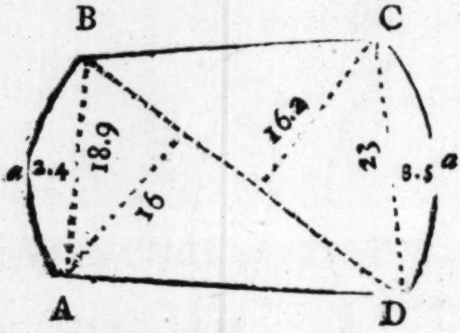
336.0425 The Area of the Segment ſought.

---

§ XIV. *Of* COMPOUND FIGURES.

MIXED or Compound Figures, are ſuch as are compoſed of rectilineal and curvilineal Figures together.

To find the Area of ſuch mixed Figures, you muſt find the Area of the ſeveral Figures, of which the whole compound Figure is compoſed, and add all the Areas together, and the Sum will be the Area of the whole compound Figure.



16.2 } add.
16.

32 Sum.

16.1 Half.
34 Diagonal.

644
483

547.4 Area of the (Trapezium

10.236 Half the Arch-line A*a*B.
14.83 Semidiameter of the Arch A*a*B.

30708
81888
40944
10236

151.79988 Area of the Sector.

From 14.83 Semidiameter.
Subtract 3.4 Verſed Sine.

Rem. 11.43 Perpend. of the Triangle.
9.45 Half the Chord AB.

5715
4572
10287

108.0135 The Area of the Triangle ſubtracted.
151.7999 From the Area of the Sector.

43.7864 The Area of the Segment A*a*BA.

	12.19	Half the Arch-line C*c*D.
	20.64	Semidiameter.
	4876	
	7314	
	24380	
	251.6016	The Area of the Sector.
From	20.64	the Semidiameter.
Subtract	3.5	Verſed Sine.
Remaind.	17.14	Perpendic of the Triangle.
	11.5	Half the Chord DC.
	8570	
	1714	
	1714	
Subtr.	197.110	Area of the Triangle.
From	251.602	The Area of the Sector.
Rem.	54.492	The Area of the Segment C*c*DC.
	43.786	The Area of the Segment A*a*BA.
	547.4	The Area of the Trapezium.
Sum	645.678	The Area of the Whole.

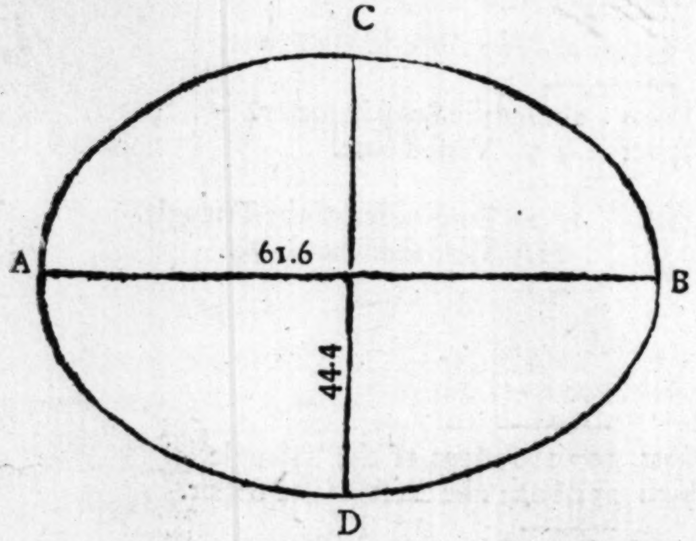
§. XV. *Of an* ELLIPSIS.

AN Ellipsis, or Oval, is a Figure bounded by a regular Curve Line, returning into itſelf; but of its two Diameters, cutting each other in the Center, one is longer than the other, in which it differs from the Circle. To find the Area thereof, this is

The

The R U L E.

Multiply the tranſverſe Diameter by the Conjugate, and multiply that Product by .7854, this laſt Product is the Area of the Oval.



61.6 The tranſverſe Diam.
44.4 The conjugate Diam.

2464
2464
2464

2735.04 The Rectangle.
.7854 Area of Unity.

1094016
1367520
2188032
1914528

2148.100416 the Area of the Oval.

Demonſtration.

If you circumſcribe any *Ellipſis* with a Circle, and ſuppoſe an infinite Number of Chord lines drawn therein, all parallel to the conjugate Diameter, as thoſe in the following Figure, then it will be,

As DA the Diameter of the Circle is : to N*n* the conjugate Diameter of the *Ellipſis* :: ſo is B*a*B any Chord in the Circle, : to *bab*, its reſpective Ordinate in the *Ellipſis*.

For, according to the Property of the Circle,

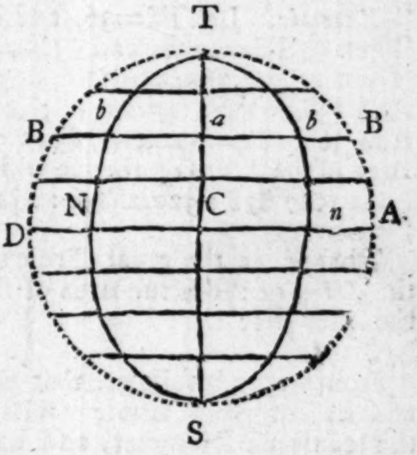
it is	1	$aS \times Ta = \square Ba$.
And		by the Property of the Ellipſis.
it is	2	$\square TC : \square NC :: aS \times Ta : \square ba$.
1, 2,	3	$\square TC : \square NC :: \square Ba : \square ba$.
3, hence	4	$TC : NC :: Ba : ba$.
Conſeq.	5	$TC : 2NC :: 2\,Ba : 2\,ba$.
That is,	6	$DA : Nn :: BaB : bab$.

But the Sum of the infinite Series of ſuch Chords as B*ab* do conſtitute the Area of the Circle. And the Sum of the like Series of their reſpective Ordinates, as *bab*, do conſtitute the Area of the *Ellipſis*.

Therefore, As TS, : to N*n* :: Circle's Area : to the *Ellipſis* Area. But TS : N*n* :: □TS : TS × N*n*; whence it follows, that,

As □ TS : Circle's Area :: ſo is TS × N*n* : *Ellipſis* Area.

Conſequently, As 1 : to .7854 :: ſo is the Rectangle, or Product of the tranſverſe and conjugate Diameter of any *Ellipſis* to its Area.



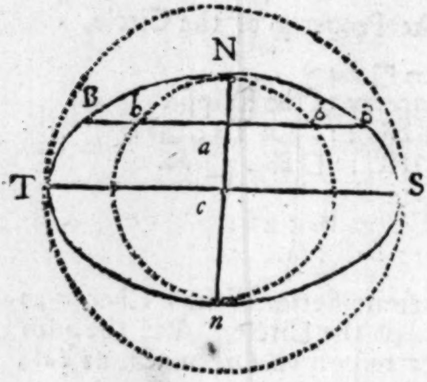
Hence it is eaſy to conceive, that the Square Root of the Product of the tranſverſe and conjugate Diameters; will be the Diameter of a Circle equal to the *Ellipſis*.

Hence alſo the Segments of an *Ellipſis*, and its circumſcribing Circle (whoſe Baſes are parallel to the conjugate Diameter, and of the ſame Height) are in Proportion one to another as their Baſes are; that is,

As B*a*B : *bab* :: Area Segment BTB : Area Segment *b*T*b*.

Or, As TS : N*n* :: Area Segment BTB : Area Segment *b*T*b*.

The Area of every *Ellipſis* is a mean Proportional between the Areas of its circumſcribing and inſcribed Circles:



The Truth of this may be eaſily deduced from the laſt; for it's already proved, that $\square TS : TS \times Nn :: $ circumſcribing Circle's Area : *Elipſis* Area.

But $\square TS : TS \times Nn :: TS \times Nn : \square Nn$. Therefore *Elipſis* Area : inſcribed Circle's Area $:: TS \times Nn : \square Nn$.

Example. Let $TS = 36$, and $Nn = 18.4$.
Then $\square TS = 1296$, and $\square Nn = 338.56$.
Then $1296 \times .7854 = 1017.8784$ great Circle's Area;
And $338.56 \times .7854 = 265.905$. *&c.* leſſer Circle's Area;
And $36 \times 18.4 = 662.4 \times .7854 = 520.24896$, which is the Area of the *Ellipſis*; then it will be,

As $1017.878 : 520.24896 :: 520.24896 : 265.905024$.

That is, as the great Circle's Area : is to the Area of the *Ellipſis* :: ſo is the Area of the *Ellipſis* to the Area of the leſſer Circle.

From hence it follows, that all Segments of an *Ellipſis* and its inſcribed Circle (whoſe Baſes are parallel to the tranſverſe Diameter, and have the ſame Height are in Proportion one to another, as the Area of the *Ellipſis* and Circle are.

That is, As the Area of the Circle : is to the Area of the *Ellipſis* :: ſo is the Segment *b*N*b* : to the Segment BNB.

Or, $Nn : TS ::$ Area Segment *b*N*b* : Area Segment BNB.

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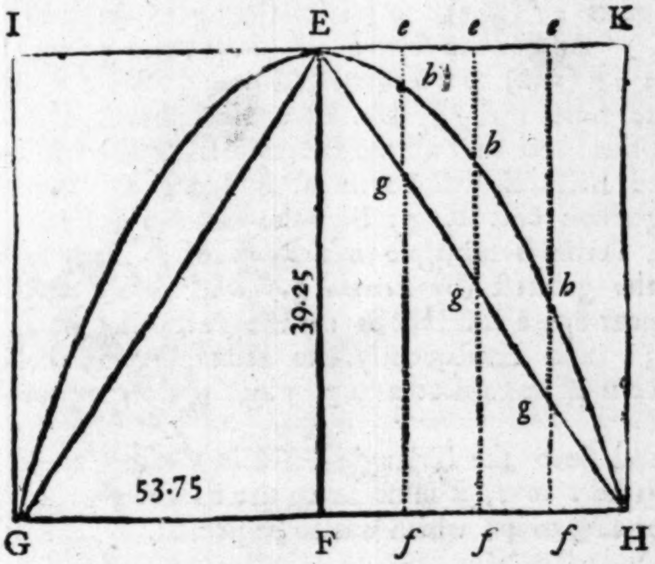
§ XVI. *Of a* PARABOLA.

A PARABOLA is a curvilinear Figure, made by the Section of a Cone, being cut by a Plane parallel to one of its Sides

Every Parabola is two Thirds of its circumſcribing Parallelogram; therefore to find the Area thereof, this is

The RULE.

Multiply the Baſe, or greateſt Ordinate, by the Perpendicular Height, and multiply that Product by 2, and divide the laſt Product by 3, the Quotient will be the Area of the Parabola.



53.75 the Ordinate GH.
39.25 the Perpendic. EF.

26875
10750
48375
16125

2109.6875
2

3) 4219.3750

1406.4583 the Area.

Demonſtration. Let FH, the Semiordinate, be divided into four equal Parts, or into 8. 16, *&c.* and thro' the Diviſions draw Lines, as *e f*, *e f*, *&c.* parallel to the Axis EF. Suppoſe alſo EF to be 4.

Then I ſay, the parabolick Space E*b*HF is to the Parrallelogram EKFH as 2 to 3: but to the Triangle EFH as 4 to 3.

For, firſt, *g f*, *g f*, *g f*, *&c.* are in continual arithmetical Proportion from the Nature of plain Triangles.

Secondly, *f e* : *g e* : : *g e* : *h e*; but *h e* in the Axis EF $=0$, and in the firſt Parallel *e f* it muſt be equal to $\frac{1}{4}$, in the next *e f* it muſt be equal to $\frac{4}{4}$, in the third to $\frac{9}{4}$, and ſo on, in a duplicate arithmetical Progreſſion.

For as *e f* $(=4)$: to *g e* $(=1)$: : *g e* $(=1)$: to *e h* $(=\frac{1}{4}.)$ And as the ſecond *e f* $(=4)$: to *e g* $(=2)$: : ſo is *e g* $(=2)$ to *e h* $(=\frac{4}{4})$ *&c.* And thus it will be if the Lines F *f*, *f f*, *&c.* be again biſected, *&c. ad infinitum.* So that all the Indiviſibles of the trilinear Space EKH*h*E will be in a duplicate arithmetical Progreſſion encreaſing: But the Sum of a Rank of ſuch Terms is ſubtriple to a Rank of as many equal to the greateſt (by *Lemma* 3.) wherefore the whole trilinear Space EKH*h*E is to the Parallelogram as 1 to 3; and conſequently the remaining parabolick Space muſt be to it as 2 to 3, which was to be proved.

And ſince the Triangle FEH is to the Parallelogram as 1 to 2, it muſt be to the Parabola as $1\frac{1}{2}$ to 2, or as 3 to 4; which was to be proved.

Before I proceed to the Menſuration of ſolid Bodies, I will lay down ſuch *Lemmas* as will be neceſſary to facilitate the Demonſtration of all ſuch Solids.

LEMMA I.

In any Series of equal Numbers, (repreſenting Lines or other Quantities) as 1, 1, 1, 1, *&c.* or 2, 2, 2, 2, *&c.* or 3, 3, 3, 3, *&c.* if one of the Terms be multiplied into the Number of Terms, the Product will be the Sum of all the Terms in the Series.

LEMMA

LEMMA II.

If a Series of Numbers, in arithmetical Progreſſion, begin with a Cypher, and the common Difference be 1, as 0, 1, 2, 3, *&c.* (repreſenting a Series of Lines or Roots begining with a Point) if the laſt Term be multiplied into the Number of Terms, the Product will be double the Sum of all the Series.

That is, putting L = the laſt Term, N = the Number of Terms, and S = the Sum of all the Series. Then will $NL = 2S$; conſequently, $\frac{1}{2}NL = S$, *viz.* one half of ſo many times the greateſt Term as there are Numbers of Terms in the Series.

Thus, $\left\{\frac{0+1+2+3+4}{4+4+4+4+4} = \frac{10}{20=NL.}\right.$ the Sum $= \frac{1}{2}NL.$

LEMMA III.

If a Series of Squares, whoſe Sides or Roots are in arithmetical Progreſſion, beginning with a Cypher, *&c.* be infinitely continued, the laſt Term being multiplied into the Number of Terms, will be triple to the Sum of all the Series, *viz.* $NLL = 3S$; or $\frac{1}{3}NLL = S$.

That is, the Sum of ſuch a Series will be one Third of the laſt, or greateſt Term, ſo many times repeated as there are Numbers of Terms in their Series.

Inſtances in ſquare Numbers.

1. $\left\{\frac{0+1+4}{4+4+4} = \frac{5}{12} = \frac{1}{3} + \frac{1}{12}\right.$

2. $\left\{\frac{0+1+4+9}{9+9+9+9} = \frac{14}{36} = \frac{7}{18} = \frac{1}{5} + \frac{1}{18},\right.$

3. $\left\{\frac{0+1+4+9+16}{16+16+16+16+16} = \frac{30}{80} = \frac{3}{8} = \frac{9}{24} = \frac{1}{3} + \frac{1}{24}.\right.$

From theſe Inſtances it is evident, that as the Number of Terms in the Series do increaſe, the Fraction or Exceſs above one Third does decreaſe, the ſaid Exceſs always being $\frac{1}{6N-6}$, which, if we ſuppoſe the Series to be infinitely continued, will quite vaniſh, and become nothing at all.

LEMMA IV.

If a Series of Cubes, whoſe Roots are in arithmetical Progreſſion, beginning with a Cypher, *&c.* (as above) be infinitely continued, the Sum of all the Series will be one Fourth NLL=S.

That is, one Fourth of the laſt Term ſo many times repeated as there are Numbers of Terms.

Inſtances in Cube Numbers.

If 0, 1, 2, 3, 4, 5, *&c.* be the Roots of the Cubes.

$$1 \left\{ \frac{0+1+8+27}{27+27+27+27} = \frac{36}{108} = \frac{4}{12} = \frac{1}{4} + \frac{1}{12} \right.$$

$$2 \left\{ \frac{0+1+8+27+64}{64+64+64+64+64} = \frac{100}{320} = \frac{10}{32} = \frac{5}{16} = \frac{1}{4} + \frac{1}{16}. \right.$$

$$3 \left\{ \frac{0+1+8+27+64+125}{125+125+125+125+125+125} + \frac{225}{750} = \frac{45}{150} = \right.$$

$$(\frac{6}{20} = \frac{1}{4} + \frac{1}{20}.$$

From theſe *Examples* it plainly appears, that as the Number of Terms in the Series increaſes, the Fraction or Exceſs above one Fourth decreaſes, the Exceſs being always $\frac{1}{4\,N-4}$ which, if we ſuppoſe the Series to be infinitely continued, will become infinitely ſmall, or nothing.

LEMMA

LEMMA V.

If a Series of Biquadrats, whoſe Roots are in arithmetical Progreſſion, beginning with a Cypher, &c. as before, be infinitely continued, the Sum of all the Terms in ſuch a Series will be one Fifth NLLLL.

The Truth of this may be manifeſt by the like Proceſs, as in the foregoing *Lemmas*, and ſo on for higher Powers.

LEMMA VI.

The Sum of an infinite Progreſſion, whoſe greateſt Term is a ſquare Number, the others decreaſing by odd Numbers, *viz.* 1, 3, 5, &c. is in ſubſeſquialtern Proportion of the Sum of the like Number of equal Terms, that is, as 2 to 3.

Inſtances in ſuch Progreſſions.

$$1\left\{\frac{9+8+5}{9+9+9}=\frac{22}{27}=\frac{2}{3}+\frac{4}{9}.\right.$$

$$2\left\{\frac{16+15+12+7}{16+16+16+16}=\frac{50}{64}=\frac{2}{3}+\frac{11}{32}.\right.$$

$$3\left\{\frac{25+24+21+16+9}{25+25+25+25+25}=\frac{95}{125}=\frac{2}{3}+\frac{7}{27}.\right.$$

$$4\left\{\frac{36+35+32+27+20+11}{36+36+36+36+36+36}=\frac{161}{216}=\frac{2}{3}+\frac{17}{72}.\right.$$

From theſe *Example* it plainly appears, that as the Number of Terms in the Series increaſes, the Fraction or Exceſs above $\frac{2}{3}$ decreaſes; and if we ſuppoſe the Series to be infinitely continued, that Exceſs will quite vaniſh, and the Sum of the infinite Series will be $\frac{2}{3}$ of ſo many equal to the greateſt.

CHAP. II.

Menſuration of SOLIDS.

SOLID Bodies are ſuch as do conſiſt of Length, Breadth, and Thickneſs; as Stone, Timber, Globes, Bullets, *&c.*

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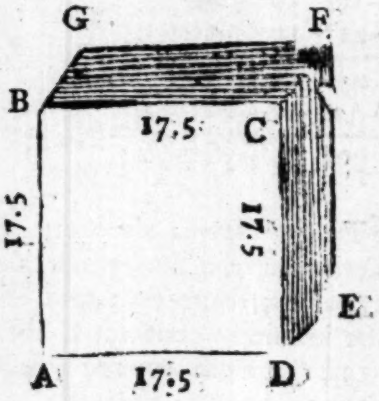
§ I. *Of a* CUBE.

A CUBE is a ſquare Solid, comprehended under ſix geometrical Squares, being in the Form of a Dye. To find the ſolid Content, this is

The RULE.

Multiply the Side of the Cube into itſelf, and that Product again by the Side, the laſt Product will be the Solidity, or ſolid Content of the Cube.



17.5

17.5
17.5
875
1225
175
306.25
17.5
153125
214375
30625

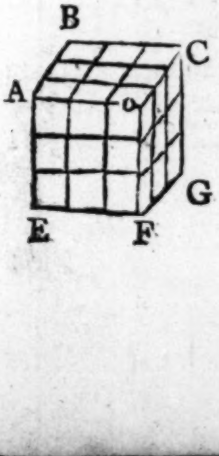
5359.375 Solid Content of the Cube.

Suppoſe ABCDEFG a cubical Piece of Stone or Wood, each Side thereof being 17 Inches and an half; multiply 17.5 by 17.5, and the Product is 306.25; which being multiplied by 17.5, the laſt Product is 5359.375, which is 5359 ſolid Inches, and 375 Parts. To reduce the ſolid Inches to Feet, divide by 1728 (becauſe ſo many cubical Inches is a Foot) and the ſolid Feet in the Cube will be 3, and 175 cubical Inches remain.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 17,5, that Extent turned over twice from 17.5, will reach to 5359, the ſolid Content in Inches. Then extend the Compaſſes from 1728 to 1, that Extent turned the ſame Way from 5359, will reach to 3.1 Feet.

DEMONSTRATION.



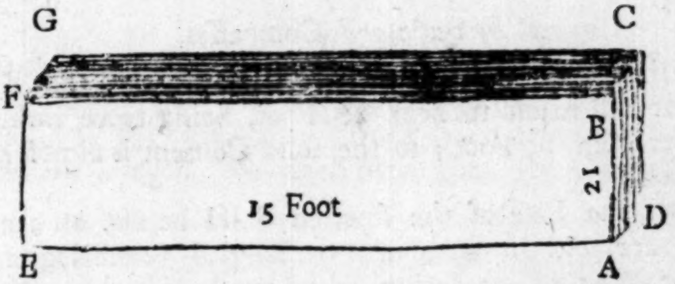
If the Square ABCD be conceived to be moved down the Plain ADEF, always remaining parallel to itſelf, there will be generated, by ſuch a Motion, a Solid having ſix Plains, the two oppoſite whereof will be equal and parallel to each other; whence it is call'd a Parallelopipedon, or ſquare Priſm. And if the Plain ADEF be a Square equal to the generating Plain ABCD, then will the generated Solid be a Cube. From hence ſuch Solids may be conceived to be conſtituted of an infinite Series of equal Squares, each equal to the Square ABCD; and AE or DF will be the Number of Terms. Therefore, if the Area of ABCD be multiplied into the Number of Terms, AE the Product is the Sum of all the Series, (*per Lemma* I.) and conſequently the Solidity of the Parallelopipedon or Cube. Or, if the Baſe ABCD, being divided into little ſquare Areas, be multiplied into the Height AE, divided by a like Meaſure for Length, after this Way you may conceive as many little Cubes to be generated in the whole Solid, as is the Number of the little Areas of the Baſe multiplied by the Number of Diviſions the Side AE contains. Thus, if the Side of the Baſe AB be 3, that multiplied into itſelf is 9, which is the Area of the ſquare Baſe ABCD; then if AE be likewiſe 3, multiply 9 by 3, and the Product is 27; and ſo many little Cubes will this Solid be cut into, if you conceive it to be cut as the Lines direct.

From this Demonſtration it is very plain, that if you multiply the Area of the Baſe of any Parallelopipedon into its Length or Height, that Product will be the ſolid Content of ſuch a Solid.

§ II. *Of a* PARALLELOPIPEDON.

LET ABCDEFG be a Parallelopipedon, or ſquare Priſm, repreſenting a ſquare Piece of Timber or Stone, each Side of its ſquare Baſe ABCD being 21 Inches, and its Length AE 15 Foot.

First



Firſt, then, multiply 21 by 21, the Product is 441, the Area of the Baſe in Inches; which multiplied by 180, the Length in Inches, and the Product is 79380, the ſolid Content in Inches. Divide the laſt Product by 1728, and the Quotient is 45.9, that is, 45 ſolid Feet, and 9 Tenths of a Foot. Or thus, Multiply 441 by 15 Feet, and the Product is 6615; divide this by 144, and the Quotient is 45.9, the ſame as before.

Or thus, by multiplying Feet and Inches.

Multiply 1 Foot 9 Inches by 1 Foot 9 Inches, and the Product is 3 Feet, 0 Inches, 9 Parts; this multiplied again by 15 Feet, gives 45 Feet, 11 Inches, 3 Parts that is, 45 Feet, and $\frac{11}{12}$ of a Foot, and $\frac{1}{4}$ of $\frac{1}{12}$.

```
              See the Work of all theſe,           F.  I.
      21                441                        1   9
      21                 15                        1   9
     ----               ----                      --------
      21               2205                        1   9
     42                441                         1   3   9
     ----               ----                      ----------
     441           144)6615(45.9                   3   0   9
     180               ----                       15
     ----               855                       ----------
   35280               1350                       45   0   0
   441                 ----                            7   6
   -----                 54                            3   9
1728)79380(45.9                                   ----------
     6912                                         45  11   3
     -----
     10260
      8640
     -----
     16200
     15552
     -----
       648
```

By Scale *and* Compaſſes.

Extend the Compaſſes from 12 to 21, and that Extent will reach to near 46 Foot, being twice turned over from 15 Foot; ſo the ſolid Content is almoſt 46 Foot.

If the Baſe of the ſquared Solid be not an exact Square, but in form of a rectangled Parallelogram, the Way of meaſuring it is much the ſame; for, firſt you muſt find the Area of the Baſe, by multiplying the Breadth by the Depth, and then multiply that Area by the Length of the Piece, as before; thus,

If a Piece of Timber be 25 Inches broad, 9 Inches deep and 25 Feet long, how many ſolid Feet are contained therein?

```
          25
           9          F. I.
        ----          2  1
         225          0  9
          25         ---------
        ----          1  6  9
        1125         25
        450          ---------
        ----         25  0  0
  144) 5625 (39      12  6  0
       432            1  0  6
       ----           0  6  3
       1305          ---------
       1296          39  0  9
       ----
         09  Anſw. 39 Feet.
```

By Scale *and* Compaſſes.

Firſt, find a mean geometrical Proportional between the Breadth and the Depth; which to do upon the Line of Numbers, you muſt divide the Space upon the Line, between the Breadth and Depth, into two equal Parts; that middle Point will be the mean Proportional ſought; thus the middle Point between 25 and 9 is at 15; ſo is 15 a mean Proportional between 9 and 25;

for, as 9 : 15 : : 15 : 25; ſo a Piece of Timber of 15 Inches ſquare is equal to a Piece 25 Inches broad and 9 Inches deep. So then, if you extend the Compaſſes from 12 to 15, that Extent turned twice over from 25 Feet, the Length, will reach to 39 Feet, the Content.

---

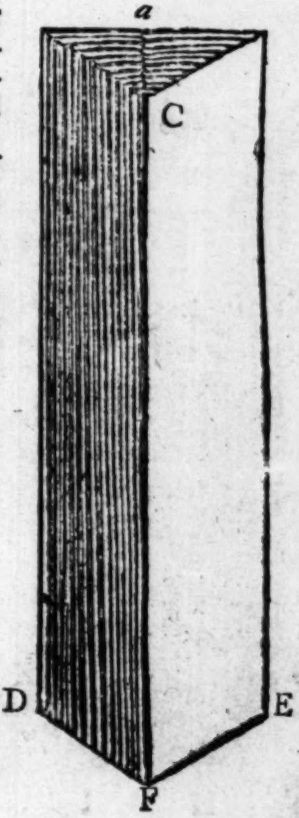
§ III. *Of a Triangular* PRISM.

A Priſm is a Solid contained under ſeveral Plains, and having its Baſes alike, equal and parallel: The ſolid Content of a Priſm (whether triangular or multangular is found by multiplying the Area of the Baſe into the Length or Height, and the Product is the ſolid Content.

Let ABCDEF be a triangular Priſm, each Side of the Baſe being 15.6 Inches, the Perpendicular thereof C*a* is 13.51 Inches, and the Length of the Solid 19.5 Feet.

Multiply the Perpendicular of the Triang. 13.51 by half the Side 7.8, and the Product is 105.378, the Area of the Baſe; which multiply by the Length 19.5, and the Product is 2054.871, which divide by 144, and the Quotient is 14.27 Feet, *fere*, the ſolid Content.

13.51	144)2054.87 (1427
7.8	144
10808	614
9457	576
105.378	388
19.5	288
526890	1007
948402	1008
105378	
2054.8710	



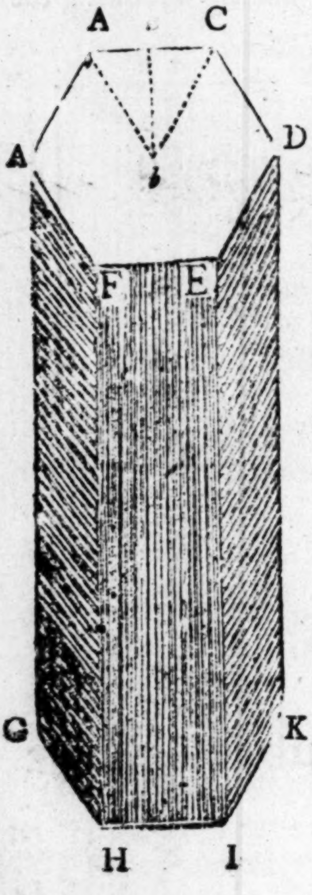
By

By Scale *and* Compaſſes.

Firſt, find a mean Proportional between the Perpendicular and half Side, (as before taught) by dividing the Space upon the Line, between 13.51 and 7.8, into two equal Parts; ſo ſhall ye find the middle Point between them to be at 10.26, which is the mean Proportional ſought: By this Means the triangular Solid is brought to a ſquare one, each Side being 10.26 Inches.

Then extend the Compaſſes from 12 to 10.26, that Extent turn'd twice downwards from 19.5 Feet, the Length, will at laſt fall upon 14.27, which is 14 Feet, and a little above a Quarter.

Let ABCDEFGHIK repreſent a Priſm whoſe Baſe is a Hexagon, each Side thereof being 16 Inches, and the Perpendicular, from the Center of the Baſe to the Middle of one of the Sides (*ab*) is 13.84 Inches, and the Length of the Priſm is 15 Feet; the ſolid Content is required.



Multiply half the Sum of the Sides 48 by 13.84, and the Product is 664.32, the Area of the hexagonal Baſe, (by § 8. *p.* 86.) which multiply by 15 Feet, the Length, the Product is 9964.8, which divided by 144, the Quotient will be 69.2 Feet, the ſolid Content required.

13.84
48
11072
5536
664.32 Area of Baſe.
15
332160
66432
144) 9964.80 (69.2
864
1324
1296
288
288
. . .

By Scale *and* Compaſſes.

Firſt, find a mean Proportional between the Perpendicular and half the Sum of the Sides, that is, divide the Space between 13.84 and 48, and the middle Point will be 25.77, then extend the Compaſſes from 12 to 25.77, that Extent will reach (being twice turned over) from 15 Feet, the Length, to 69.2 Feet, the Content.

To find the ſuperficial Content of any of the forementioned Solids, you muſt take the Girt of the Piece. and multiply by the Length, and to that Product add the two Areas of the Baſes, the Sum will be the whole ſuperficial Content. *Example* of the Hexagonal Priſm laſt mentioned: The Sum of the Sides being 96, and the Length 15 Feet, that is, 180 Inches, which multiplied by 96, the Product is 17280 ſquare Inches, to which add twice 664.32, the Areas of the two Baſes, and the Sum is 18608.64, the Area of the Whole, which is 129.22 Feet.

180
96
1080
1620
17280
664.32
664.32
18608.64

144) 18608.64 (129.22
420
1328
326
384
96

The ſuperficial Content of the whole Solid is 129.22 Feet.

By Scale *and* Compaſſes.

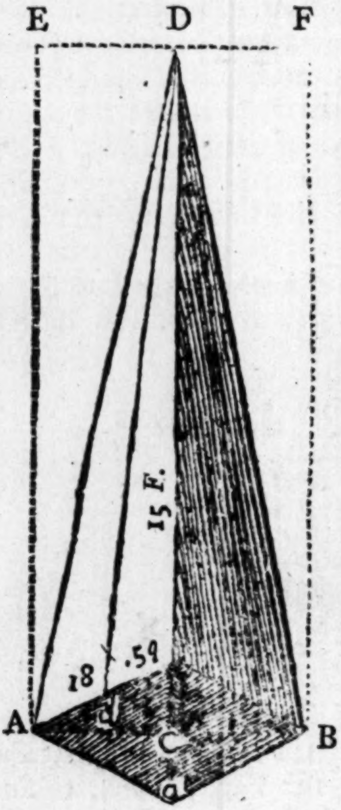
Extend the Compaſſes from 144 to 180, that Extent will reach from 96 to 120 Feet: Then, to find the Area of the Baſe, extend the Compaſſes from 144 to 13.84, that Extent will reach from 48 to 4.6 Feet; add 120 Feet, and twice 4.5 Feet, and it makes 129.2 Feet, the ſuperficial Content, as before.

The Demonſtration of theſe laſt Solids will be the ſame as in the firſt Section; for as in that, ſo in theſe, the Area of the Baſe is multiplied into the Length to find the Content, and the ſame Reaſon is given for one as for the other.

§ IV.

§ IV. *Of a* PYRAMID.

A Pyramid is a ſolid Figure, whoſe Baſe is a Polygon, and whoſe Sides are plain Triangles, their ſeveral Tops meeting together in one Point. To find the ſolid Content thereof, this is



The RULE.

Multiply the Area of the Baſe by a third Part of the Altitude, or Length, and the Product is the ſolid Content of the Pyramid.

Let ABD be a ſquare Pyramid, each Side of the Baſe being 18.5 Inches, and the Perpendicular Height CD is 15 Feet: Multiply 18.5 by 18.5, and the Product is 342.25, the Area of the Baſe in Inches: which multiplied by 5, a third Part of the Height, and the Product is 1711.25: this divided by 144, the Quotient is 11.88 Feet the ſolid Content.

18.5
18.5
925
1480
185
342.25 Area of the Baſe.
5
144)1711.25(1188 Content.

F.	I.	Pts.	
1	6	6	
1	6	6	
1	6	6	
	9	3	
		9	3
2	4	6	3
			5
11	10	7	3

By Scale *and* Compaſſes.

Extend the Compaſſes from 12 to 18.5 Inches, that Extent turned twice over from 5 Feet a third Part of the Height) will fall at laſt upon 11.88 Feet, the ſolid Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe 37, and the Product is 6668.88, which divided by 144, the Quotient is 46.31 Feet, the ſuperficial Content of all but the Baſe; then to that add 2.38 Feet, the Baſe, and it makes 48.69 Feet, the whole ſuperficial Content.

180.24 The ſlant Height *d*D.
37
———
126168
54072
———
144)6668.88(46.31
576 2.38
——— ———
908 48.69 The whole Content.
864
———
448
432
———
168
144
———
24

144)342.25(2.38
288
———
542
432
———
1105
1152
———

By Scale *and* Compaſſes.

Extend the Compaſſes from 144 to 180.24, that Extent will reach from 37 to 46.31 Feet, the Area of the four Triangles; and extend the Compaſſes from 144 to 18.5 (one Side of the Baſe) that Extent will reach from 18.5 to 2.38 *ferè*; which added to the other, the Sum is 48.69, the whole Superficies.

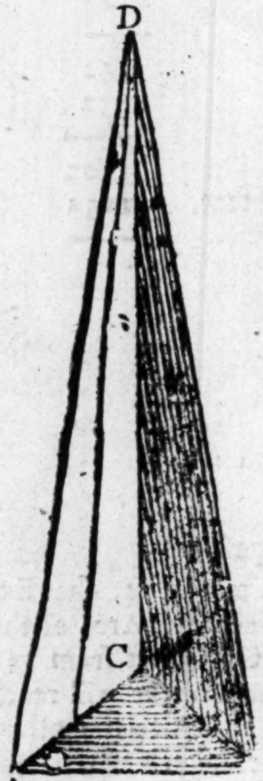
DEMON-

DEMONSTATION,

Every Pyramid is a third Part of the Priſm, that hath the ſame Baſe and Height, (by *Euclid*, 12. 7.)

That is the ſolid Content of the Pyramid ABD (in the laſt Figure) is one third Part of its circumſcribing Priſm ABEF,

For every Pyramid that hath a ſquare Baſe (ſuch as A*a*B*b* in the laſt Figure) is conſtituted of an infinite Series of Squares, whoſe Sides or Roots are continually increaſing in arithmetical Progreſſion, beginning at the Vertex or Point D, its Baſe A*a*B*b* being the greateſt Term, and its perpendicular Height CD is the Number of all the Terms: But the laſt Term multiplied into the Number of Terms, the Product will be triple the Sum of all the Series, (by *Lemma* III.) conſequently $\frac{NLL}{3}=S$. And S is equal to the ſolid Content of the Pyramid From hence it will be eaſy to conceive, that every Pyramid is $\frac{1}{3}$ of its circumſcribing Priſm, (that is, of a Priſm of equal Baſe and Altitude) what Form ſoever its Baſe is of, *viz.* whether it be ſquare, triangular, pentagular, *&c.* You may very ealſiy prove a triangular Pyramid to be a third Part of a Priſm of equal Baſe and Altitude, by cutting a triangular Priſm of Cork, and then cut that Priſm into three Pyramids, by cutting diagonally, as I have ſeveral times done, to ſatisfy myſelf and others.



Let ABCD be a triangular Pyramid, each Side of the Baſe being 21.5 Inches, and its perpendicular Height 16 Feet; the Content ſolid and ſuperficial is required.

Firſt, find the Area of the Baſe, by multiplying half the Side by the Perpendicular let fall from the Angle of the Baſe to the oppoſite Side; which Perpendicular will be found to be 18.62; the Half thereof is 9.31, which multiplied by

21.5,

21.5, the Product is 200.165 Inches, the Area of the Baſe. Then, becauſe the Altitude 16 cannot exactly be divided by 3, therefore I take the third Part of 200.165, which is 66.72, and multiply it by 16, and the Product is 1067.52, which divided by 144, the Quotient is 7.41 Feet, the ſolid Content.

			F.	I.	Ps.	
9.31	Half the Perpen.					
21.5	The Side.	Side.	1	9	6	
		Half Perpen.		9	4	
4655						
931			1	4	1	6
1862					7	2
3)200.165	Area Baſe.	Area Baſe.	1	4	8	8
						4
66.72	A third Part.					
16	Height.		5	6	10	8
						4
40032						
6672		3)	22	3	6	8
144)1067.52(7.41	Sol. Content.	Cont.	7	5	2	2
1008						
595						
576						
192						
144						
48						

In caſting this up by Feet and Inches, inſtead of multiplying by 16, the Height, I break 16 into two ſuch Numbers, as being multiplied together the Product may be 16, *viz.* into 4 and 4, and firſt multiply by one, and then the other; a third Part of the laſt Product is the Content.

By Scale *and* Compaſſes.

Firſt, find a geometrical mean Proportional, (as before directed) by dividing the Space between 21.5 and 9.31 into two equal Parts, and you will find the middle Point at 14.15, which is the mean Proportional ſought. Then extend the Compaſſes from 12 to 14.15, that Extent (turned twice over from 16 Feet) will fall at laſt upon 22.23 ; a third Part thereof is 7 41 Feet, the Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe, and to that Product add the Area of the Baſe, the Sum is the whole ſuperficial Content.

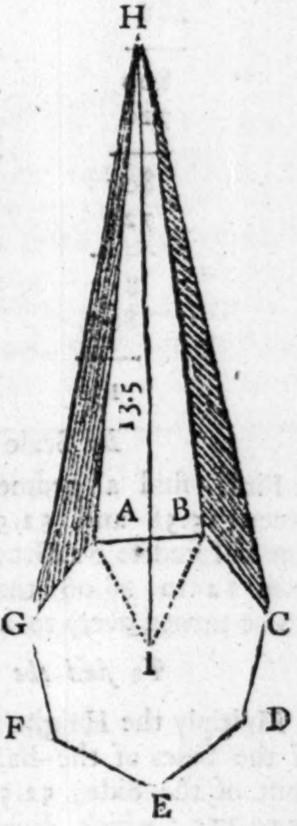
```
                 192.1 Inches, the ſlant Height dD.
Half Periphery   32.25=21.25+10.75
                 -----
                 9605
                3842
               3842
              5763
              -------                              [Baſe.
              6195.225 Inches, the Area of all but the
               200.165 Area of the Baſe add.
              -------
          144)6395.390(44.41 Feet, the whole Content.
              576
              ---
               635
               576
               ---
                593
                576
                ---
                 179
                 144
                 ---
                  35
```

By Scale *and* Compaſſes.

Extend the Compaſſes from 144 to 192.1, that Extent will reach from 32.25 (half the Periphery of the Baſe) to 43.02 Feet, the Content of the upper Part.

And extend the Compaſſes from 144 to half the Perpendicular 9.31, that Extent will reach from the Side 21.5 to 1.39 Feet, the Area of the Baſe, which added to the other makes 44.41 Feet, the Content of the Whole.

Let ABCDEFGH be a Pyramid, whoſe Baſe is a Heptagon, each Side thereof being 15 Inches, and the Perpendicular of the Heptagon is 15.58 Inches, and the perpendicular Height of the Paramid HI is 13.5 Feet; the Content ſolid and ſuperficial is required.



Multiply 15.58 (the Perpendicular) by 52.5, half the Sum of the Sides of the Heptagon, and the Product is 817.95, which multiplied by 4.5, *viz.* $\frac{1}{3}$ of the Height, and the Product is 3680.775.

Then divide this laſt Product by 144, and the Quotient is 25.56 Feet, the Content.

15.58 The Heptagon's Perpendicular.
52.5 The half Sum of the Sides.

7790
3116
7790

817.950
4.5 A third Part of the Height.

4089750
3271800

144) 3680.7750 (25.56 Solid Feet.
288

800
720

807
720

877
864

13

By Scale *and* Compaſſes.

Firſt, find a geometrical mean Proportional between 15.58 and 52.5 (as is before directed) which you will find to be 28.06; then extend the Compaſſes from 12 to 28.06, that Extent will reach from 4.5 (twice turned over) to 25.56 Feet.

To find the ſuperficial Content.

Multiply the Height taken from the Middle of one of the Sides of the Baſe 162.75 Inches, by the half Sum of the Sides 52.5 Inches, and the Product is 8544.375; which divided by 144, the Quotient is 59.335 Feet, the Content of the upper Part,

162.75

```
  162.75
    52.5
  ------
   81375
  32550
 81375
 -------
144)8544.375 (59.335 Feet.
                5.68 Baſe add.
   ----       ------
   1344       65.015
    483
     517
      855
   ------
      135
```

```
144)817.95(5.68
      979
      1155
     ------
         3
```

By Scale *and* Compaſſes

Extend the Compaſſes from 144 to 162.75, that Extent will reach from 52.5 to 59.333 Feet.

And extend the Compaſſes from 144 to 15.58, the Perpendicular of the Heptagon, the Extent will reach from 52.5 to 5.68 Feet, the Content of the Baſe; which add to the former, the Sum is 65.015, the whole Superficial Content.

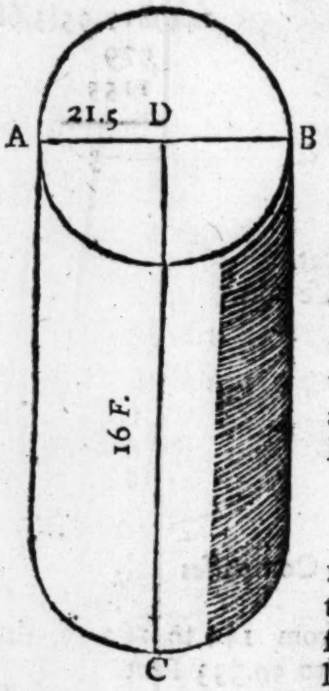
§ V. *Of a* CYLINDER

A CYLINDER is a round Solid, having its Baſes circular, equal and parallel, in Form of a Rolling Stone uſed in Gardens. To find the ſolid Content thereof, this is

The RULE,

Multiply the Area of the Baſe by the Length and the Product is the ſolid Content.

Let



Let ABC be a Cylinder, whoſe Diameter AB is 21.5 Inches, and the Length CD is 16 Foot; the ſolid Content is required.

Firſt, ſquare the Diameter 21.5, and it makes 462.25; which multiply by .7854, and the Product is 363.05115. Then multiply this by 16, and the Product is 5808.8164. Divide this laſt Product by 144, and the Quotient is 40.34 Feet, the ſolid Content.

By Scale *and* Compaſſes

Extend the Compaſſes from 13.54 to 21.5, the Diameter, that Extent (turned twice over from 16, the Length) will at laſt fall upon 30.34, the ſolid Content.

To find the ſuperficial Content.

Firſt, (by Ch. 1. Sect. IX. Prob. 2.) find the Circumference of the Baſe 67.54, which multiplied by 16, the Product is 1080.64; which divided by 12, the Quotient is 90.05 Feet, the Curve Surface; to which add 5.04 Feet, the Sum of the two Baſes, and the Sum is 95.09 Feet, the whole ſuperficial Content.

67.54
16

40524
6754

12)1080.64

90.05

90.05 }
5.04 } add:

95.09

363.05
2

144)726.10(5.04

610

34

By

By Scale *and* Compaſſes.

Extend the Compaſſes from 12 to 67.54, (the Circumference) that Extent will reach from 16, the Length, to 90.05 Feet, the Curve-Surface.

And extend the Compaſſes from 12 to 21.5, (the Diameter) that Extent (turned twice from .7854) will at laſt fall upon 2.52 Feet, the Area of one Baſe; which doubled is 5.04; this added to the Curve-Surface makes 95.09 Feet, the whole ſuperficial Content.

DEMONSTRATION.

The ſolid Content of every Cylinder is found by multiplying the Area of its Baſe into its Height, as aforeſaid: For every right Cylinder is only a round Priſm, being conſtituted of an infinite Series of equal Circles; that of its Baſe, or End, being one of the Terms, and its Height CD (in the former Figure) is the Number of all the Terms: Therefore the Area of its Baſe AB being multiplied into CD, will be its Solidity (by *Lemma* I.) Let D=AB, and H=CD

Then .7854 DD × H=its Solidity.

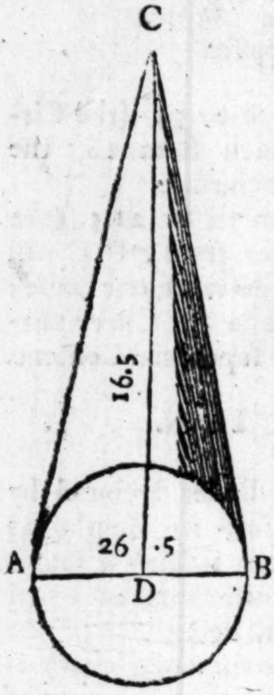
---

§ VI. *Of a* CONE.

A CONE is a Solid, having a circular Baſe, and growing ſmaller and ſmaller till it ends in a Point, which is called the Vertex, and may be nearly repreſented by a Sugar-Loaf. To find the Solidity thereof, this is

The RULE.

Multiply the Area of the Baſe by a third Part of the perpendicular Height, and the Product is the ſolid Content.



Let ABC be a Cone, the Diameter of whoſe Baſe AB is 26.5 Inches, and the Height of the Cone DC is 16.5 Feet. Firſt, ſquare the Diameter 26.5, and it is .702.25 ; which multiply by .7854, and the Product is 551.54715 ; which multiply by 5.5, and the Product is 3035.47825 ; which divided by 144, the Quotient is 21.07 *fere*, the ſolid Content of the Cone.

26.5 The Diameter.
26.5
1325
1590
530
702.25 The Square.
.7854
280900
351125
561800
491575
551.54|715 Area of the Baſe.
5.5 A third Part of the Height.
275770
275770
144)3033.47|0(21.06 Feet, the Content.
153
947
83

By Scale *and* Compaſſes.

Extend the Compaſſes from 13.54 to 26.5, (the Diameter) that Extent turned twice over from 5.5 (a third Part of the Height) will at laſt fall upon 21.06 Feet, the Content.

To find the ſuperficial Content.

Multiply half the Circumference 41.626 by the ſlant Height AC 198.46, the Product is 8261.09596; which divided by 144, the Quotient is 57,37, *fere*, the Curve-Surface; to which add the Baſe, the Sum is 61.2, the ſuperficial Content.

41.626 The half Circumference of the Baſe.
198.46 Inches, the ſlant Height.

249756
166504
333008
374634
41626

144)8261.09596(57.37 Feet, *fere.*
1061 3.83 The Baſe add.
530 61.20 The whole Content.
989
144)551.54(383
1195
434
2

By Scale *and* Compaſſes.

Extend the Compaſſes from 144 to 198.46, that Extent will reach from 41.626 to 57.37 Feet, the Curve-Surface.

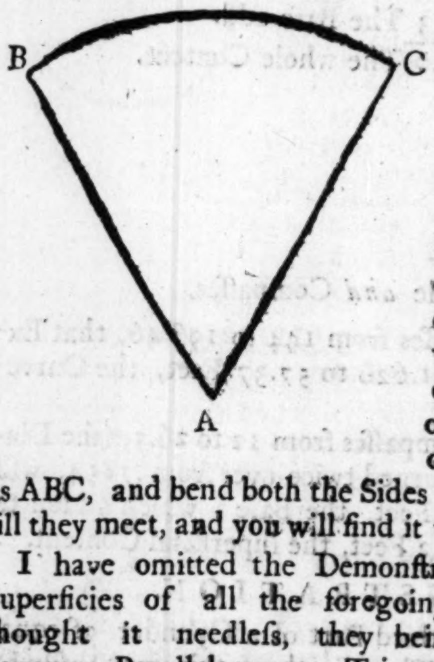
And extend the Compaſſes from 12 to 26.5, (the Diameter) that Extent, turned twice over from .7854, will at laſt fall upon 3.83 Feet, the Baſe; which added to 57.37, the Sum is 61.2 Feet, the ſuperficial Content.

DEMONSTRATION.

Every Cone is the third Part of a Cylinder of equal Baſe and Altitude. The Truth of this may eaſily be conceived by only conſidering, that a Cone is but a round Pyramid, and therefore it muſt needs have the ſame Ratio to its circumſcribing Cylinder, as the ſquare Pyramid hath to its circumſcribing Parallelopipedon, *viz.* as 1 to 3. However, to make it yet clearer, let it be farther conſidered, That

Every right Cone is conſtituted of an infinite Series of Circles whoſe Diameters do continually increaſe in arithmetical Progreſſion, beginning at the Vertex, or Point C, the Area of its Baſe AB being the greateſt Term, and its perpendicular Height DC the Number of all the Terms; therefore the Area of the Circle of the Baſe, multiplied by a third Part of the Altitude DC, will be the Sum of all the Series, equal to the Solidity of the Cone, by *Lemma* III.

The Curve-Superficies of every right Cone, is equal to half the Rectangle of the Circumference of its Baſe into the Length of its Side.



For the Curve-Surface of every right Cone is equal to the Sector of a Circle, whoſe Arch BC is equal to the Periphery of the Baſe of the Cone, and Radius AB equal to the ſlant Side of the Cone. Which will appear very evident, if you cut a Piece of Paper in the Form of a Sector of a Circle, as ABC, and bend both the Sides AB and AC together till they meet, and you will find it to form a right Cone.

I have omitted the Demonſtrations touching the Superficies of all the foregoing Solids, becauſe I thought it needleſs, they being all compoſed of Squares, Parallelograms, Triangles, *&c* which Figures are all demonſtrated before. And if the Area of all ſuch Figures as compoſe the Solid be found ſeverally, and added together, the ſum will be the ſuperficial Content of the Solid.

§ VII.

§ VIII. *Of the Fruſtum of a* PYRAMID.

A Fruſtum of a Pyramid, is the remaining Part when the Top is cut off by a Plane parallel to the Baſe. To find the ſolid Content thereof, there are ſeveral Rules.

RULE I.

To the Rectangle (or Product) of the Sides of the two Baſes add the Sum of their Squares; that Sum being multiplied into one third Part of the Fruſtum's Height, will give its Solidity, if the Baſes be ſquare.

Or thus, which is the ſame in Effect.

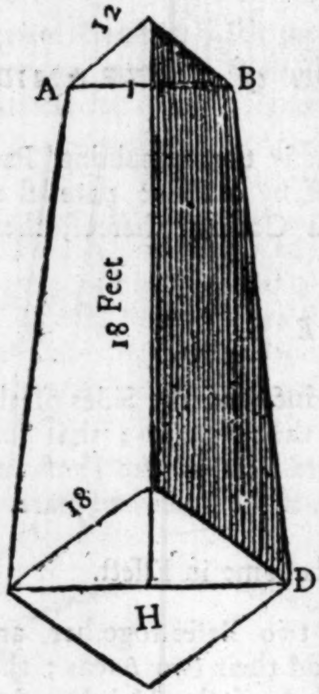
Multiply the Areas of the two Baſes together, and to the ſquare Root thereof add their two Areas; that Sum multiplied by one Third of the Height, gives the Solidity of any Fruſtum, ſquare or multangled.

RULE II.

To the Rectangle of the Sides of the two Baſes, add one third Part of the Square of their Difference; that Sum being multiplied into the Height, will produce the Solidity, if the Baſes be ſquares: But if they be triangular or multangular, the Rectangle of the Sides, with the third Part of the Square of their Difference, will be the ſquare of a mean Side; and the ſquare Root thereof will be ſuch a mean Side as will reduce the tapering Solid to a Priſm equal thereunto.

Example, Let ABCD be the Fruſtum of a ſquare Pyramid, the Side of the greater Baſe 18 Inches, and the Side of the leſſer 12 Inches, and the Height 18 Foot; the Solidity thereof is required.

First,



Firſt, multiply the two Sides together, 18 by 12, and the Product is 216, and the Difference of the Sides is 6, whoſe Square is 36; a third Part thereof is 12, which added to 216, the Sum is 228 Inches, the Area of a mean Baſe; which multiplied by 18 Feet, the Length, the Product is 4104; this divided by 144, the Quotient is 28.5 Feet, the Content.

Or, by the firſt Rule, thus; the Square of 18 is 324, and the Square of 12 is 144, and the Rectangle of 18 by 12 is 216; the ſum of theſe three 684, which multiplied by 6, the Product is 4104; which divided by 144, the Quotient is 28.5 Feet, the ſame as before.

See the **Work** of both Ways.

```
   18           6 Diff.          18          12
   12           6                18          12
  ----        ----              ----        ----
  216      3) 36 Square      { 324 Square   144 Square
   12 add     ----           { 144
  ----     12 a Third        { 216
  228 the Sum                  ----
   18 the Height               684 the Sum.
  ----                           6 a 3d of the Height.
 1824                          ----
  228                     144) 4104 (28.5 Feet.
  ----                         ----
144) 4104 (28.5                1224
  ----                          720
 1224                          ----
  720                           ...
  ----
  ...
```

By Feet and Inches thus :

	F.	I.	I.
Multiply	1	6	6
by	1		6
Product	1	6	3)36 q.
add	0	1	12
Multiply	1	7	
by	18	0 Height.	
	18	0	
	9	0	
	1	6	
Content	28	6	

Or thus,

F.	I.	
2	3	Square of the greater
1	6	The Rectangle
1	0	Square of the leſs.
4	9	Trip. of a mean Area
6	0	A 3d of the Height
28	6	

To find the ſuperficial Content.

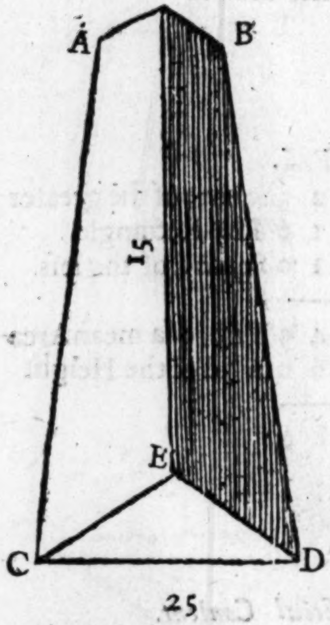
The Perimeter of the greater Baſe is 72. and the Perimeter of the leſſer Baſe is 48 ; add both the Perimeters together, the Sum is 120 ; the Half thereof is 60 ; which multiplied by 18 Feet, the Product is 1080 ; this divided by 12, the Quotient is 90 Feet ; to which add the two Baſes 2.25 Feet, and 1 Foot, the Sum is 93.25 Feet, the whole ſuperficial Content.

18
4
72
48
2) 120
60

12
4
48

18 The Height.
60
12)1080
90 Feet.
2.25 The greater Baſe
1 The leſſer Baſe.
93.25 Sum.

Again.



Again. Let ABCD be the Fruſtum of a triangular Pyramid, each Side of the greater Baſe 25 Inches, and each Side of the leſſer Baſe 9 Inches, and the Length 15 Feet; the ſolid Content thereof is required.

By the ſecond Rule; multiply 25 by 9, and the Product is 225; and the Difference between 25 and 9 is 16; which ſquared makes 256, a third Part thereof is 85.333, which added to 225, and the Sum is 310.333: this multiplied by .433, the Product is 134.374, *&c.* which is the Area of a mean Baſe: and that multiplied by 15 Feet, the Length, the Product is 2015.610, which divided by 144, the Quotient is 13.99 Feet, the Solidity.

Or thus, by the latter Part of the firſt Rule: Find the Area of the greater Baſe, which you will find to be 270.625, and the Area of the leſſer Baſe will be 35.073; theſe two Areas multiplied together, the Product is 9491.630625, the ſquare Root thereof is 97.425, to which add the two Areas, and the Sum is 403.123; which multiplied by a third Part of the Length 5, and the Product is 2015.610; and that divided by 144, and the Quotient is 13.99 Feet, as before.

See the Working of both.

25 25
9 9
Product 225 16 Difference
16
96
16
3) 256 the Square
85.333 a third Part
225. add.
310.333
.433 Tabular Numb. (*vide p.* 95)
930999
930999
1241332
134.374189 mean Area.
15 Length.
671870945
134374189
144)2015.61|2835(13.99 Feet.
575
1436
1401
105

25
25
125
50
625 Square.
.433
1875
1875
2500
270.625 Areas

9
9
81 Square

.433
81
433
3464
35.073

270.625
35.073
811875
1894375
1353125
811875
9491.630625(97.425
81
187)1391
1309
1944)8263
7776
19482) 48706
38964
194845)974225
974225
......

270.625

```
270.625 Greater Area.
 97.425 The mean Proportional.
 35.073 The leſſer Area.
-------
403.123 The Triple of a mean Area.
      5 A third Part of the Height.
-------
144)2015.615(13.99 Feet, the Solidity.
     575
      1436
      1401
      ----
       105
```

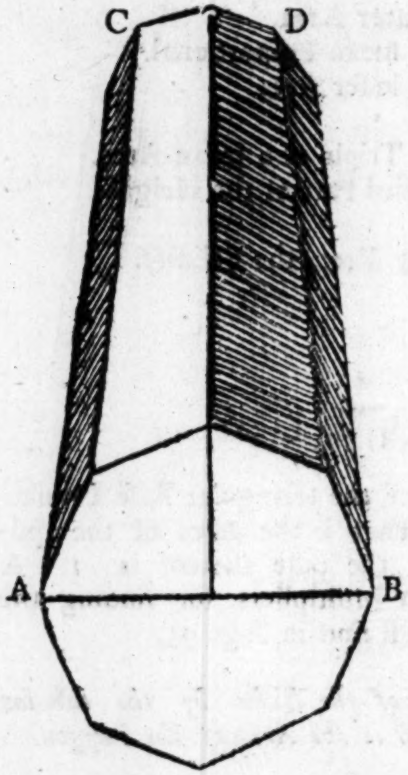
In finding the Area of the triangular Baſe I multiplied by .433, becauſe that is the Area of the equilateral Triangle when the Side thereof is 1. A Table of the Areas, or Multipliers, for finding the Areas of Polygons, you'll find in Page 95.

Multiply the Square of the Sides by the tabular Number, and the Product is the Area of the Polygon.

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 75, and the Perimeter of the leſſer Baſe is 27; the Sum of both is 102, and the half Sum is 51, which multiplied by 15 Feet, the Product is 765; which divided by 12, the Quotient is 63.75; to which add the Sum of the two Baſes 2.12 Feet, and the Sum is 65.87 Feet, the whole ſuperficial Content.

NOTE, That 51 ſhould have been multiplied by the ſlant Height, but the Difference it would make is but .06 of a Foot, which is inconſiderable.



Again. Suppoſe A B C D to be the Fruſtum of a Pyramid, having an Octagonal Baſe, each Side thereof being 9 Inches, and each Side of the leſſer Baſe 5 Inches, and the Length or Height 10.5 Feet; the Solidity is required.

By the ſecond Rule. Multiply the greater Side 9 by the leſſer Side 5, and the Product is 45; then the Difference between 9 and 5 is 4, which ſquared makes 16; a third Part thereof is 5.3333, which added to 45, the Sum is 50.3335, multiply this laſt by the Number in the Table, 4.8284, and the Product is 243.0292, the Area of a mean Baſe; which multiplied by the Height 10.5 Feet, and the Product is 2551.8066; then divide this laſt Product by 144, and the Quotient is 17.72 Feet, the ſolid Content.

See the Work.

See the Work.

Mult. 9 Inches 9 ſrom the greater Side.
by 5 Inches 5 ſubtract the leſſer.

Prod. 45 4
4

3) 16 ſquared.

5 3333 a third Part.
Add 45

Sum 50.3333 the Square of mean Side.
4.8284 Tabular Number, Page 95.

2013332
402666
10067
4026
201

243.0292 a mean Area.
10.5 the Height.

12151460
2430292

144)2551.80|660(17.72
144

1111
1008

1038
1008

300
288

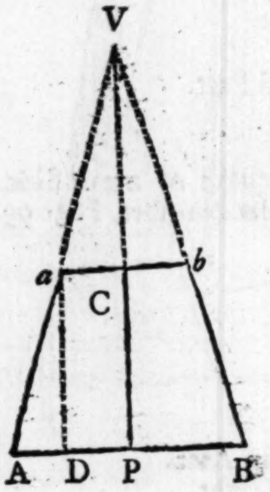
12

To find the ſuperficial Content

The Perimeter of the greater Baſe is 72, and the Perimeter of the leſſer Baſe is 40, and their Sum is 112; the Half thereof is 56, which multiplied by the Height 10.5 Feet, and the Product is 588; which divided by 12, the Quotient is 49 Feet: to which add the Sum of the two Baſes 3.55, and the Sum is 5255 Feet, the whole ſuperficial Content.

DEMONSTRATION.

From the Rules delivered in the fourth and ſixth Sections, the two foregoing Rules may eaſily be demonſtrated.



Suppoſe a ſquare Pyramid, as ABV, to be cut by a Plane at *a b*, parallel to its Baſe AB, and it were required to find the Solidity of the Fruſtum or Part *a b* AB. Let there be given

$D=BA$, the Side of the greater Baſe.
$d=ba$, the Side of the leſſer Baſe.
$H=CP$, the Perpendicular Height

Firſt, | 1 | $D-d : H :: d : \frac{dH}{D-d} = VC$ by the Figure
Then | 2 | $DD \times \frac{H+VC}{3} =$ the whole Pyramid BVA by Section the fourth.
And | 3 | $dd \times \frac{1}{3} VC =$ the Pyramid *aVb* cut off.

Then, in the 2d and 3d Steps, if inſtead of VC you take $\frac{dH}{D-d} =$ to it by the firſt Step, it will be,

viz. 1.2	4	$\frac{DDDH}{3D-3d}$ = the whole Pyramid BVA.
and 1.3.	5	$\frac{dddH}{3D-3d}$ = the Pyramid *aVb*.
4—5.	6	$\frac{DDDH-dddH}{3D-3d}$ = the Fruſtum *ai*AB.

And by dividing DDD—ddd by D—d, and then multiplying the Quotient by $\frac{1}{3}$ H, the laſt Step will be reduced to DD+Dd+dd : × $\frac{1}{3}$ H = the Fruſtum *ab*AB ; which in Words is thus ;

To the Rectangle of the Sides of the two Baſes, add the Sum of their Squares ; that Sum being multiplied into one Third of the Fruſtum's Height, will give its Solidity, which is the ſame as the firſt Rule of this Section.

See the Work of the Diviſion.

D—d) DDD—ddd (DD+Dd+dd
DDD—DDd

DDd—ddd
DDd—Ddd

Ddd—ddd
Ddd—ddd

0 0

The ſame Reaſon will hold good for all Fruſtums of Pyramids or Cones, whether the Baſe be triangular or multangular, becauſe the Squares of the Sides of any Figure, or the Squares of the Diameters of Circles are proportional to the Area, which proves the latter Part of the firſt ſaid Rule.

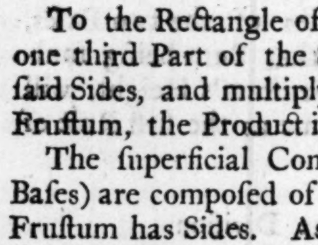
Again, to prove the ſecond Rule.

Suppoſe	1	$x=D-d$. And $F=$the Fruſtum.
then	2	$DD+Dd+dd=\frac{3F}{H}$ by the laſt.
1 ⊙ 2	3	$xx=DD-2Dd+dd$.
2—3	4	$3Dd=\frac{3F}{H}-xx$.
4÷3	5	$Dd=\frac{F}{H}\frac{1}{3}xx$. Or $Dd+\frac{1}{3}xx=\frac{F}{H}$
5×H	6	$DD+\frac{1}{3}xx:\times H=F$, the Fruſtum *ab*AB.

Which in Words is thus :

To the Rectangle of the Sides of the two Baſes add one third Part of the Square of the Difference of the ſaid Sides, and multiply the Sum by the Height of the Fruſtum, the Product is the Solidity of the Fruſtum.

The ſuperficial Content of Fruſtum (all but the Baſes) are compoſed of Trapeziums, ſo many as the Fruſtum has Sides. As the ſquare Fruſtum *ab*AB, in the laſt Figure, is compoſed of four Trapeziums, having the two upper and alſo the two lower Angles equal ; if therefore the Trapezium *ab*AB be cut in two by the Line CP, and the two Pieces laid together, the Line *b*B upon the Line *a*A, the narrow End of the one to the broad End of the other, it will form a right angled Parallelogram, as is plain by the Figure annexed; the Parallelogram DCEP being equal to the Trapezium *ab*AB becauſe the Side D*a* is equal to PB, and EA is eqaul to *a*C. Therefore, to find the Area of the Trapezium, add half the Sides *ab* to half the Side AB, and it makes DC or EP ; which multiply by the Height PC, the Product is the Area of the Parallelogram DCEP, equal to the Trapezium *ab* AB ; then, if that be multiplied



tiplied by the Number of Trapeziums, the Product will be the ſuperficial Content of the Fruſtum, wanting the Baſes. Or if the whole Perimeter of the greater Baſe be added to the Perimeter of the leſſer Baſe and half the Sum multiplied by the Height, the Product will be the ſuperficial Content of all Trapeziums at once.

NOTE, That half the Sum of the Perimeters ſhould be multiplied by the ſlant Height, up the Middle of one of the Trapeziums; but in the foregoing *Examples* I have multiplied by the perpendicular Height, becauſe the Difference is very inconſiderable,

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§ VIII. *Of the Fruſtum of a* CONE.

A Fruſtum of a Cone is that Part which remains, when the Top End is cut off by a Plane Plarallel to the Baſe. To find the ſolid Content, the Rules are the ſame Effect as for the Fruſtum of a Pyramid.

RULE I.

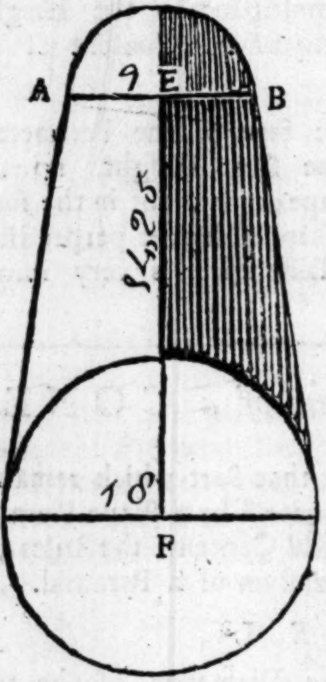
To the Rectangle of the Diameters of the two Baſes add the Squares of the ſaid Diameters, and multiply the Sum by .7854, the Product will be the Triple of a mean Area, which multiplied by $\frac{1}{3}$ of the perpendicular Height, that Product will be the ſolid Content.

Or thus: Multiply the Areas of the greater and leſſer Baſes together, and out of the Product extract the ſquare Root, and add the two Areas and ſquare Root together, and multiply the Sum by one Third of the perpendicular Height, the Product is the ſolid Content.

RULE II.

To the Rectangle of the greater and leſſer Diameters, add one third Part of the Square of their Difference,

ference, and multiply the Sum by .7854, the Product is a mean Area; which multiplied by the perpendicular Height, the Product is the Solidity.



Example. Let ABCD be the Fruſtum of a Cone, whoſe greater Diameter CD is 18 Inches, and the leſſer Diameter AB 9 Inches, and the Length 14.25 Feet; the ſolid Content is required.

Multiply 18 by 9, and the Product is 162, and the Difference between 18 and 9 is 9, whoſe Square is 81, a third Part is 27; which add to 162, the Sum is 189; this multiplied by .7854, the Product is 148.44; which divived by 144, the Quotient is 1,03 Feet, the Area of a mean Baſe; which multiply by 14.25 Feet, the Height, the Product is 14.6775 Feet, the ſolid Content.

Or thus, by the firſt Rule.

The Square of 18 (the greater Diameter) is 324, and the Square of 9 (the leſſer Diameter) is 81, and the Rectangle, or Product of 18 by 9, is 162; the Sum of theſe three is 567, which multiplied by .7854, the Product is 445.3218; which divided by 144, the Quotient is 3.09 Feet, the triple Area of a mean Baſe; this multiplied by 4.75 Feet (a third Part of the Height) and the Product is 14.6775 Feet, the Solidity, the ſame as before.

See the Work.

	18	18 from	.7854
	9	9 ſubtract	189
	162	9 remains	70686
Add	27	9	62832
			7854
Sum	189	3) 81 Square	
			144)148.44|06(1.03
		27 a Third,	144
	Height	14.25 Feet	444
	Area Baſe	1.03 Feet	432
		4275	12
		14250	
Solid	Content	14.6775 Feet	

324 The Square of 18
162 The Rectangle.
81 The Square of 9.

567 The triple Square of a mean Diameter.

.7854
567

54978
47124
39270

144) 445.32|18 (3.09
4.75 a Third of the Height.
1332
1545
36
2163
1236

The Solidity 14.6775

To find the ſuperficial Content.

By Chap. I. Sect. IX *Problem* 2. you will find the Circumference of the greater Baſe to be 56.5488, and of the leſſer Baſe 28.2744; the Sum of both is 84.8232; the half Sum is 42.4116; which multiplied by 14.25 Feet, and the Product is 604.36, *&c*, which divide by 12, the Quotient is 50.36 Feet, the Curve-Surface; to which add the Sum of the two Baſes, 2.21 Feet, the Sum is 52.57 Feet, the whole ſuperficial Content.

---

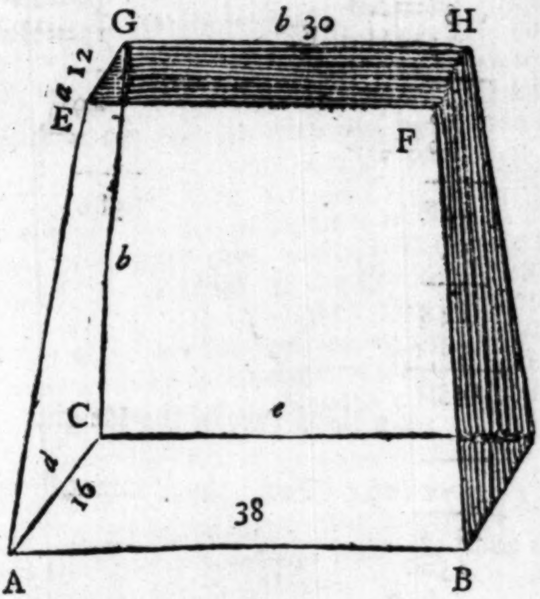
§ IX. *To meaſure the* Fruſtum *of a rectangled* Pyramid, *called a* PRISMOID, *whoſe Baſes are parallel to one another, but diſproportional.*

The RULE.

TO the greateſt Length add half the leſſer Length, and multiply the Sum by the Breadth of the greater Baſe, and reſerve the Product.

Then, to the leſſer Length add half the greater Length, and multiply the Sum by the Breadth of the leſſer Baſe, and add this Product to the other Product reſerved, and multiply that Sum by a third Part of the Height, and the Product is the ſolid Content.

Example



Example. Let ABCDEFGH be a Priſmoid given, he Length of the greater Baſe AB 38 Inches, and its Breadth AC 16 Inches ; and the length of the leſſer Baſe EF is 30 Inches, and its Breadth 12 Inches, and he Height 6 Feet ; the ſolid Content is required.

To the greater Length AB 38, add half EF the ſſer Length 15, the Sum is 53 ; which multiply by 6, the greater Breadth, and the Product is 848, hich reſerve.

Again. To EF 30, add half AB 19, and the Sum 49 ; which multiply by 12, (the leſſer Breadth EG) e Product is 588 ; to which add 848, (the reſerved oduct) and the Sum is 1436 ; which multiply by 2, third Part of the Height) and the Product is 2872 ; vide this Product by 144, and the Quotient is 19.94 et, the ſolid Content.

38=AB
15=Half EF

53
16=AC

318
53

848
588

1436
2=a third Part of the Height.

144) 2872 (19.94 Feet, the Content.
1432
1360
640
64

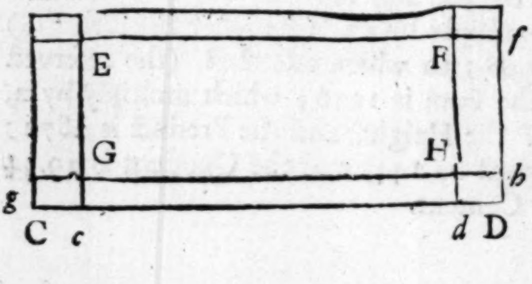
30=EF
19=Half AB

49
12=EG

588

To prove this RULE,

Let us ſuppoſe the Solid cut into Pieces, ſo as t make it capable of being meaſured by the foregoin Rules, thus: Let ABCD repreſent the greater Baſ and EFGH the leſſer Baſe; and let the Solid be ſuppoſed to be cut thro' by the Lines *ac*, *bd*, and *ef*, *g* from the Top to the Bottom; ſo will there be a Para lelopipedon having its Baſes equal to the leſſer Baſe E GH, and its Height 6 Feet equal to the Height of t Solid: Mul ply 30, (t Length of t Baſe) by 1 (the Bread thereof,) a the Product 360; whi multiplied the Height Fe



Feet, and the Product is 2160. Then there are two Wedge-like Pieces, whoſe Baſes are *ab*EF, and GH*cd*; if theſe two Pieces be laid together, the thick End of one to the thin End of the other, they will compoſe a rectangled Parallelopipedon; which to meaſure, multiply the Length of the Baſe 30, by its Breadth 2, and the Product is 60; which multiply by 6, (the Height) the Product is 360. Then there are two other Wedge-like Pieces, whoſe Baſes are *e*E*g*G, and F*f*H*h*; theſe two laid together will compoſe a rectangled Parallelopipedon; to meaſure this, multiply the Length of the Baſe 12, by the Breadth 4: the Product is 48; which multiply by 6, (the Height) the Product is 288. And *laſtly*, there are four rectangled Pyramids, at each Corner one; which to meaſure, multiply the Length of one of the Baſes 4, by its Breadth 2, the Product is 8; which multiplied by 2, (a third Part of the Height) the Product is 16, and that multiplied by 4, (becauſe there are four of them) the Product is 64. Then add all theſe together, and the Sum is 2872; and divide by 144, the Quotient is 19.94 Feet, the ſame as before; which ſhews the Rule to be true.

See the Work.

```
  12      30      12      4
  30       2       4      2
 ----    ----    ----    ---
 360      60      48      8
   6       6       6      2
 ----    ----    ----    ---
2160     360     288     16
 360                      4
 288                     ---
  64                     64
-----
144) 2872 (19.94 Feet the whole Content.
     ----
     1432
      1360
        640
     ------
        64
```

To find the ſuperficial Content.

Half the Perimeter of the greater Baſe is 54, and half the Perimeter of the leſſer Baſe is 42; which added together the Sum is 96, which multiplied by 6, (the Height) the Product is 576; divide this Product by 12, the Quotient is 48 Feet; to which add the Sum of the two Baſes 6.72 Feet, and the Sum is 54.72 Feet, the whole ſuperficial Content.

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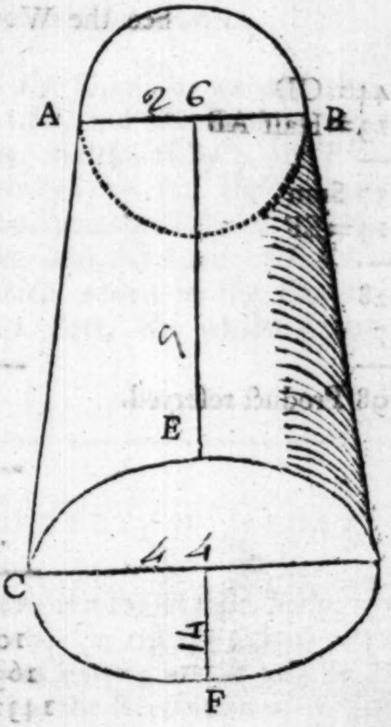
§ X. *To meaſure a* CYLINDROID; *that is, a* Fruſtum *of a* CONE *having its Baſes parallel to each other, but unlike.*

The RULE.

TO the longeſt Diameter of the greater Baſe add half the longeſt Diameter of the leſſer Baſe, and multiply the Sum by the ſhorteſt Diameter of the greater Baſe, and reſerve the Product.

Then, to the longeſt Diameter of the leſſer Baſe add half the longeſt Diameter of the greater Baſe, and multiply the Sum by the ſhorteſt Diameter of the leſſer Baſe, and add the Product to the former reſerved Sum, and that Sum will be the triple Square of a mean Diameter; which multiplied by .7854, and that Product multiplied by a third Part of the Height, the Product is the ſolid Content.

Example. Let AB CD be a Cylindroid, whoſe Bottom Baſe is an Oval, the tranſverſe Diameter being 44 Inches, and the conjugate Diameter 14 Inches; and the upper Baſe is a Circle whoſe Diameter is 26 Inches, and the Height of the Fruſtum is 9 Feet; the Solidity is required.



To 44 (the greater Diameter of the lower Baſe) add 13, half the Diameter of the leſſer Baſe, the Sum is 57; which multiplied by 14, (the conjugate Diameter of the greater Baſe) the Product is 798, which reſerve. Then to 26 (the Diameter of the leſſer Baſe) add 22, (half the tranſverſe Diameter of the greater Baſe) and the Sum is 48; which multiplied by 26, (the Diameter of the leſſer Baſe) the Product is 1248; to which add the former reſerved Product. the Sum is 2046; which multiplied by .7854, the Product is 1606.9284; which multiplied by 3, (a third Part of the Height) the Product is 4820.7852; which divided by 144, the Quotient is 33.47 Feet, the ſolid Content.

See the Work.

44=CD	26=AB
13=Half AB.	22=Half CD.
57 Sum	48 Sum
14=EF	26=AB.
228	288
57	96
798 Product reſerved.	1248
	798 Add.
	2046
	.7854
	8184
	10230
	16368
	14322
	1606.9284
	3
	144) 4820.78|52(33.47
	500
	687
	1118
	110

This Rule being the ſame with that in the laſt Section, the Proof of that may ſerve as a ſufficient Proof of this, if what has been before written be well conſidered.

To

To find the ſuperficial Content.

To the Periphery of the Ellipſis 97.41 add the Periphery of the Circle 81.68, and the Sum is 179.09; the half thereof 89.545, multiplied by 9, the Product is 805.905; which divided by 12, the Quotient is 67.16 Feet, the Curve-Surface: Then the Area of the Ellipſis is 3.36 Feet, and the Area of the Circle is 3.68 Feet; both which added to the Curve-Surface, the Sum is 74.2 Feet, the whole ſuperficial Content.

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§ XI. *Of a* SPHERE, *or* GLOBE.

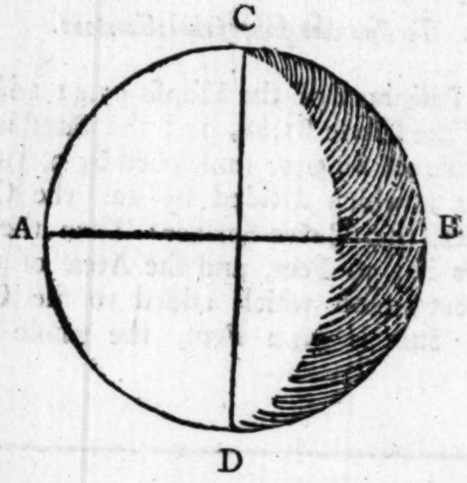
A SPHERE, or GLOBE, is a round ſolid Body, every Part of whoſe Surface is equally diſtant from a Point within it, called its Center; and it may be conceived to be formed by the Revolution of a Semicircle round its Diameter. To find its Solidity, this is

The RULE.

1. Multiply the Axis or Diameter into the Circumference, the Product is the ſuperficial Content; which, multiply by a ſixth Part of the Axis, the Product is the Solidity.

2. Or thus: As 21 : to 11 : : ſo is the Cube of the Axis to the ſolid Content.

Or, as 1 : is to .5236 : : ſo is the Cube of the Axis to the ſolid Content.



Example. Let ABCD be a Globe whoſe Axis is 20 Inches, then the Circumference will be 62.832: Then, by the firſt Rule, multiply the Circumference by the Axis, and the Product will be 1256.64, which is the ſuperficial Content in Inches; take a ſixth Part thereof, which is 209.44, (becauſe an exact ſixth Part of 20 cannot be taken) multiply that ſixth Part by 20, (the Axis) and the Product is 4188.8, the Solidity in Inches. Or, if you multiply the ſuperficial Content by the Axis, and take a ſixth Part of the Product, the Anſwer will be the ſame.

Or, thus by the ſecond Rule:

The Cube of the Axis is 8000, which multiplied by 11, the Product is 88000: which divided by 21, the Quotient is 4190.47, the Solidity.

Or, by the third Rule:

If the Cube of the Axis be multiplied by .5236, the Product is 4188.8 the Solidity, the ſame as by the firſt Way. If you divide 4188.8 by 1728, the Quotient is 2.424 Feet.

See the WORK.

62.832

62.832

20

6)1256.640 The ſuperficial Content.

209.44 A ſixth Part.

20

4188.80 The Solidity in Inches.

As 21 : 11 : : 8000

11

21) 88000 (4190.47 The Content.

40

190

100

160

13

As 1 :.5236 : : 8000

8000

1728)4188.8000(2.424 Feet, the Solidity.

7328

4160

7040

128

NOTE, If the Axis of a Globe be 1, the Solidity will be .5236 ; and if the Circumference be 1, the Solidity will be .016887.

By Scale *and* Compaſſes.

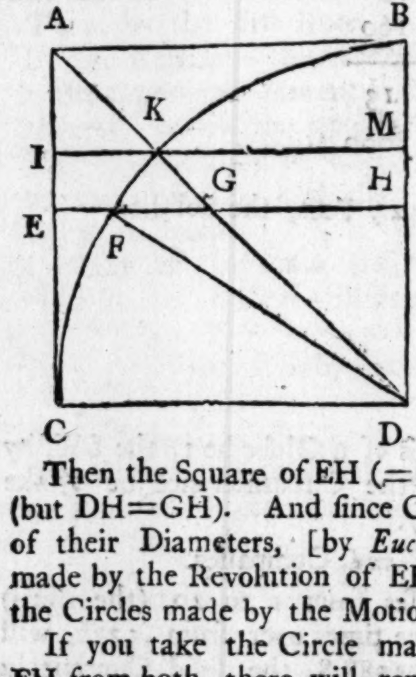
Extend the Compaſſes from 1 to 20, (the Axis) that Extent turned three times over from .5236, will at the laſt fall upon 4188.8, the ſolid Content in Inches. Or, Extend the Compaſſes from 1728 to 8000, (the Cube of the Axis) that Extent will reach from .5236 to 2.424, the ſolid Content in Feet.

Extend the Compaſſes from 1 to 20, (the Axis) that Extent turned twice over from 3.1416, will at laſt fall upon 1256.64, the ſuperficial Content in Inches. Or, Extend the Compaſſes from 144 to 400, (the Square of the Axis) that Extent will reach from 3.1416 to 8.72, the ſuperficial Content in Feet.

DEMONSTRATION.

Every Sphere is equal to a Cone whoſe perpendicular Axis is the Radius of the Sphere, and its Baſe a Plane, equal to all the Surface of it.

For you may conceive the Sphere to conſiſt of an infinite Number of Cones, whoſe Baſes, taken all together, compoſe the Surface, and whoſe Vertexes meet all together in the Center of the Sphere: Hence the Solidity of the Sphere will be gained by multiplying its Surface by $\frac{1}{3}$ of its Radius.



Let the Square ABCD, the Quadrant CBD, and the right-angled Triangle ABD, be ſuppoſed all three to revolve round the Line BD as an Axis; then will the Square generate a Cylinder, the Quadrant an Hemiſphere, and the Triangle a Cone all of the ſame Baſe and Altitude.

Then the Square of EH (= □FD) = □FH + □DH (but DH = GH). And ſince Circles are as the Squares of their Diameters, [by *Euclid*, 12. 2.) the Circle made by the Revolution of EH muſt be equal to both the Circles made by the Motions of FH and GH.

If you take the Circle made by the Revolution of FH from both, there will remain the Circle made by the Motion of GH, equal to the Ring deſcribed by the Motion of EF. And thus it will aways be, where ever you draw the Line EH or IM, &c.

Therefore the Aggregate, or Sum, of all the Rings made by the Revolution of EF's, muſt be equal to that of all the Circles made by the Motion of GH's *i, e.* the Diſh like Solid, formed by the revolving Rings, will be equal to the Cone, formed by the Revolution of the GH's, which are the Elements of the Triangle ABD; that is, the Diſh like Solid will be as the Cone, $\frac{1}{3}$ of the circumſcribing Cylinder, and conſequently the Hemiſphere muſt be $\frac{2}{3}$ of it: Wherefore the Sphere is $\frac{2}{3}$ of the Circumſcribing Cylinder.

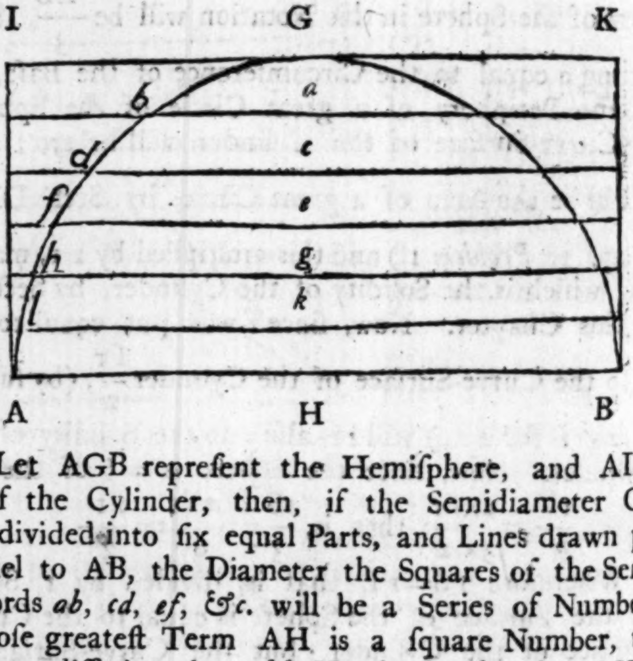
Let the Radius of the Sphere be $r = CD$, then the Diameter will be $2r$; let the Surface of the Sphere, generated by the revolving Semicircle, be called S, and that of the Cylinder, formed by the Revolution of $2AC = 2r =$ Diameter, be called ſ. Wherefore, in what was juſt now proved, the Expreſſion for the Solidity of the Sphere in this Notation will be $\frac{1rS}{3}$, and putting c equal to the Circumference of the Baſe, or for the Periphery of a great Circle of the Sphere, the Curve-Surface of the Cylinder will be $2rc$; alſo $\frac{rc}{2}$ will be the Area of a great Circle, by Sect. IX. of Chap. 1. *Problem* 1) and this multiplied by $2r$, makes $1rc$, which is the Solidity of the Cylinder, by Sect. V. of this Chapter. Now, ſince ſ was put equal to $2rc$ = to the Curve-Surface of the Cylinder $\frac{\text{ſ}r}{2}$, (by ſubſtituting ſ for $2rc$) will be alſo = to the Solidity of the Cylinder. Now, ſince the Sphere is $=\frac{2}{3}$ of the Cylinder, $\frac{rS}{3} = \frac{2\times\text{ſ}r}{3\times 2}$; that is, $\frac{rS}{3} = \frac{2\text{ſ}r}{6} = \frac{\text{ſ}r}{3}$

Wherefore $rS = r\text{ſ}$, that is, divided by r, $S = \text{ſ}$; or the Surface of the Sphere is equal to the Curve-Surface of the Cylinder, but the Curve-Surface of the Cylinder was $2rc$.

Wherefore, to find the Area of the Surface of either Sphere or Cylinder, you muſt multiply the Diameter ($=2r$) by the Circumference of a great Circle of the Sphere, or by the Periphery of the Baſe. From this Notation alſo $\frac{r\,c}{2}$, the Area of a great Circle of the Sphere, is plainly $\frac{1}{4}$ of 2 r c, the Surface of the Sphere; that is, the Surface of the Sphere is quadruple of the Area of the greateſt Circle of it.

Wherefore, to 2 r c, the Convex-Surface of the Cylinder, add r c equal to the Area of both its Baſes, you will have 3 r c; which ſhews you, that the Surface of the Cylinder (including it Baſes) is to the Surface of the Sphere as 3 to 2; or that the Sphere is $\frac{2}{3}$ of the circumſcribing Cylinder, in Area as well as Solidity.

Or you may prove the Sphere to be $\frac{2}{3}$ of the Cylinder of the ſame Baſe and Altitude, by *Lemma* VI. aforegoing, thus:



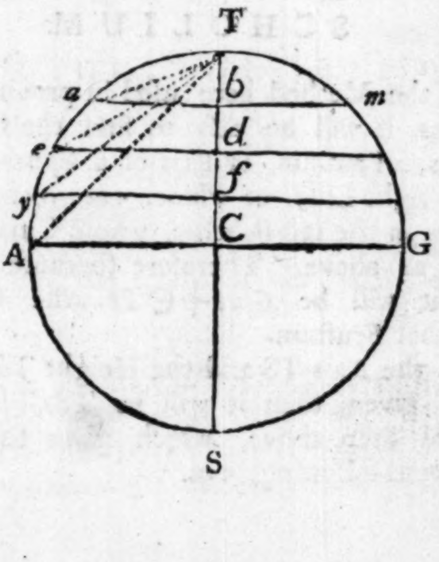
Let AGB repreſent the Hemiſphere, and AIKB half the Cylinder, then, if the Semidiameter GH be divided into ſix equal Parts, and Lines drawn parallel to AB, the Diameter the Squares of the Semichords *ab*, *cd*, *ef*, &c. will be a Series of Numbers, whoſe greateſt Term AH is a ſquare Number, the others differing by odd Numbers, that is, AH is 36, *kl* 35. *gh* 32, *ef* 27, *cd* 20, *ab* 11: But an infinite Series of ſuch Numbers are in Proportion to an infinite Number of Terms equal to the greateſt, as

2 to 3. And becauſe the Hemiſphere is compoſed of an infinite Number of Circles, whoſe Diameters are the Chords of the Semicircle, and the half Cylinder is compoſed of an infinite Number of Circles, whoſe Diameters are all equal to the Diameter of the Semicircles AB ; therefore the Hemiſphere is in Proportion to the half Cylinder as 2 to 3 ; and conſequently the whole Sphere bears the ſame Proportion to the whole Cylinder.

That the Superficies of every Sphere (or Globe) is equal to four times the Area of its greateſt Circle is thus proved :

The Solidity of the Sphere is conſtituted of an infinite Number of parallel Circles, as is aforeſaid ; conſequently the Superficies of the Sphere will be compoſed of the Peripheries of thoſe Circles which conſtituted its Solidity.

NOTE, In the following Demonſtrations, ⊙ ſignifies any Circle in general ; and if any two Letters be joined to it, thus. ⊙ AB, *&c.* then it denotes the Area of ſuch a Circle as thoſe two Letters repreſent the Radius of.



Let $D=TS$, the Axis of any Sphere, then, according to the Property of a Circle, it

will be	1	$D-Tb\times Tb=\Box ab$;
that is,	2	$D\times Tb-\Box Tb=\Box ab$;
therefore	3	$D\times Tb=\Box aT$.
For	4	$\Box ab+\Box Tb=\Box aT$, (*Euc.* 1. 47.)
And	5	$D\times Td=\Box eT$,
	6	$D\times Tf=\Box yT$.

Hence it is evident, that the Series $\Box aT$, $\Box eT$, $\Box yT$, &c. are in the ſame Ratio with Tb, Td, Tf, &c. *viz.* in arithmetical Progreſſion: Whence it follows, that the $\odot aT=$ to the Sum of all the Circle's Peripheries between T and *b*.

And $\odot eT=$ the Sum of all the Circle's Peripheries between T and *d*, &c.

Conſequently, that the $\odot AT=$ the Sum of all the Circle's Peripheries, included between T and C; that is, $\odot AT=$ the Superficies of the Hemiſphere.

And becauſe $\Box AC+\Box TC=\Box AT$, and $\Box AC$ is equal to $\Box TC$; therefore $\odot AT=2\odot AC$, is the Superficies of the Hemiſphere.

Conſequently, $4\odot AC$ will be the Superficies of the whole Sphere. Which was, &c.

SCHOLIUM.

From the Method here uſed in proving the whole Superficies, it will be eaſy to find the Curve-Superficies of any Fruſtum, or Part of a Sphere, that is cut off by a right Line, or Plane, *viz.* ſuch as the Fruſtum aTm in the laſt Scheme, whoſe Curve-Superficies is $\odot aT$, as above. Therefore (becauſe $\Box ab+\Box Tb=aT$) it will be $\odot ab+\odot Tb=$ the Curve-Superficies of that Fruſtum.

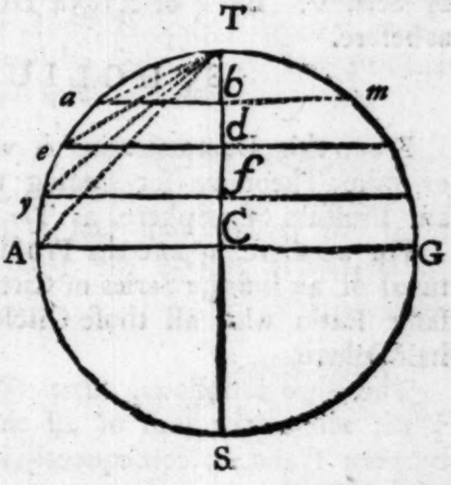
But if the Axis TS and the Height Tb of the Fruſtum are given, then it will be $TS\times Tb=aT$, as in the third Step above, which gives the Proportion or Theorem following, *viz.*

As the Axis of the Sphere : to the whole Superficies of the Sphere : : ſo is the Height of any Fruſtum : to its Curve-Superficies:

To which if there be added the Area of the Fruſtum's Baſe, the Sum will be the whole Superficies of the Fruſtum.

That the Solidity of every Sphere is two Thirds of its circumſcribing Cylinder. *may be thus proved.*

According to the Work above, it appears that ⊙*ab*, ⊙*ed*, ⊙*yf*, &*c.* do coſtitute the Solidity of the Sphere; and that □*a*T, *e*T, *y*T, &*c.* are a Series of Terms in arithmetical Progreſſion, □AT being the greateſt Term, and TC the Number of Terms; therefore ⊙AT × $\frac{1}{2}$TC = the Sum of all the Series, by *Lemma* II.



And becauſe □*a*T — □T*b* = □*ab*. □*e*T — □T*d* = □*ed*. □*y*T — □T*f* = □*yf*. □AT — □TC = □AC, &*c.* wherein □T*b*, □T*d*, □T*f*, &*c.* are a Series of Squares, whoſe Roots T*b*, T*d*, T*f*, are in arithmetical Progreſſion; □TC being the greateſt Term, and TC the Number of Terms; therefore ⊙TC × $\frac{1}{3}$ TC = the Sum of all the Series, by *Lemma* III.

Conſequently, ⊙AT × $\frac{1}{2}$TC : — ⊙TC × $\frac{1}{3}$TC = the Sum of all the Series ⊙*ab*, ⊙*ed*, ⊙*yf*, &*c.* which conſtitute the Solididy of the half Sphere ATG. Put D = 2TC, the Axis of the Sphere; then $\frac{1}{3}$ D = $\frac{1}{2}$ TC, and $\frac{1}{6}\text{D}=\frac{1}{3}\text{TC}$. And becauſe $\square\text{AT}=2\square\text{TC}$, therefore $\odot\text{AT}=2\odot\text{TC}=1.5708\ \text{DD}$; and $1.5708\ \text{DD}\times\frac{1}{4}\text{D}=0.3927\ \text{DDD}$.

Again, $\odot\text{TC}\times\frac{1}{3}\text{TC}=0{,}7854\ \text{DD}\times\frac{1}{6}\text{D}=1309\ \text{DDD}$, then $0.3927\ \text{DDD}-0.1309\ \text{DDD}.=0.2618\ \text{DDD}$, the Solidity of the half Sphere.

Conſequently, $0.2618\ \text{DDD}\times 2=.5236\text{DDD}$, will be the ſolid Content of the whole Sphere, which is equal to $\frac{2}{3}$ of the Cylinder; the Diameter of whoſe Baſe, and alſo its Height, is $=\text{D}$.

For $0.7854\ \text{DDD}=$ the Solidity of the Cylinder, by Sect. V. But $\frac{2}{3}$ of $0.7854\ \text{DDD}=0.5236\ \text{DDD}$, as before.

SCHOLIUM.

From this Demonſtration it will be eaſy to deduce or raiſe Theorems for finding the ſolid Content of any Fruſtum of a Sphere, as T*m* in the laſt Figure.

For we there ſuppoſe the Fruſtum *a*T*m* to be conſtituted of an infinite Series of Circles, which have the ſame Ratio with all thoſe Circles that conſtitute the half Sphere.

Therefore it follows, that $\odot a\text{T}\times\frac{1}{2}\text{T}b:-\odot b\text{T}\times\frac{1}{3}\text{T}b$, will be the Sum of all the Circle's intercepted between T and *b*; conſequently it will be the Solidity of that Fruſtum.

And becauſe $\square ab+\square\text{T}b=a\text{T}$, therefore $\odot ab+\odot\text{T}b\times\frac{1}{2}\text{T}b:-\odot\text{T}b\times\frac{1}{3}\text{T}b=$ the Solidity. Let $c=ab$, half the Diameter of the Fruſtum's Baſe; $h=\text{T}b$ its Height, and $\text{S}=$ the Solidity of the Fruſtum. Then $\odot ab=3.141600$, and $\odot\text{T}b=3.1416\ hh$; conſequently,

$$\frac{3.1416\ cch+3.1416\ hhh}{2}-\frac{3.1416\ hhh}{3}=\text{S};$$

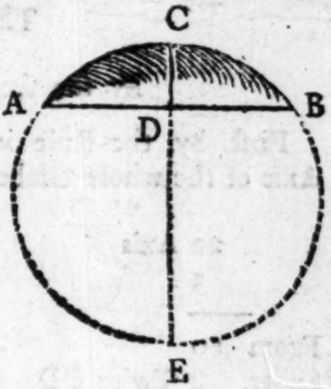
which being reduced, will become $3cch+hh\times 0.5236=\text{S}$: Which is one Theorem for finding the Solidity of the Fruſtum, and may be expreſſed in Words thus:

If to three times the Square of the Semidiameter of the Fruſtum's Baſe you add the Square of the Height of the Fruſtum, and multiply the Sum by the Height of the Fruſtum, and that Product multiplied by .5236, the Product will be the ſolid Content.

But if the Axis of the Sphere, and the Height of the Fruſtum be given, then put D= the Axis, h=the Height of the Fruſtum, and c as before, it will be $\overline{D-h}\times h=cc$, *viz.* $Dh-hh=cc$. Then will $3Dhh-2hhh=3cch+hhh$; conſequently $\overline{3Dhh-2hhh}\times 0.5236=S$, the Fruſtum's Solidity; which is another Theorem for finding the Solidity of the Fruſtum, and may be expreſſed in Words thus:

From three times the Axis ſubtract twice the Height of the Fruſtum, and multiply the Remainder by the Square of the Height, and that Product multiply by .5236, this laſt Product will be the Solidity of the Fruſtum.

Example. Let ABCD be the Fruſtum of a Sphere ſuppoſe AB (the Diameter of the Fruſtum's Baſe) be 16 Inches, and CD (the Height) 4 Inches; the Solidity is required.



By the firſt Rule.

8
8
64 Square of the Semidiameter AD.
3
192
16 Add the Square of CD.
208
4 Multiply by CD
832

.5236
832
10472
15708
41888
The Solidity 435.6352

By the ſecond Rule, thus.

Firſt, by the Rule in Page 117, you will find the Axis of the whole Globe to be 20 Inches.

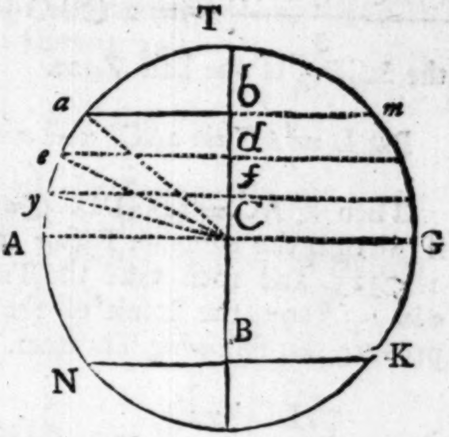
20 Axis
3
From 60
Subtr. 8 Twice CD.
Rem. 52
Mult. 16 Square of CD.
312
52
Prod. 832

.5236
832
10472
15708
41888
435.6352

The Solid Content the ſame as before.

And if it be required to find the middle Part *am* NK, uſually called the middle Zone of a Sphere.;

Then becauſe it is ſuppoſed that $am = NK$, or (which is all one) that $bC = CB$. therefore it is plain, that if twice the Segment aTm be taken from the whole Sphere there, will remain the middle Zone *am* NK.



But becauſe that Work is a little troubleſome, I will here ſhew how to raiſe a Theorem for the doing it.

Firſt, becauſe $AC = yC = eC = aC = TC$, therefore it will be $\square AC - \square Cf = \square yf$, $\square AC - \square Cd = \square ed$ $\square AC - \square Cb = \square ab$, &c.

Here, becauſe $\square AC$, $\square AC$, $\square AC$, *&c* are a Series of Equals, and Cb the Number of all the Terms; therefore $\square AC \times Cb =$ the Sum of all that Series, *per Lemma* I.

And $\square Cf$, $\square Cd$, $\square Cb$, &c. being a Series of Squares, whoſe Roots are in arithmetical Progreſſion, beginning at the Center C, *viz.* 0. Cf, Cd, Cb, &c. wherein the greateſt Term is $\square Cb$, and the Number of Terms is Cb; therefore $\square Cb \times \frac{1}{3} Cb =$ the Sum of all the Series, *per Lemma* III.

Conſequently, the $\odot AC \times Cb. - \odot Cb \times \frac{1}{3} Cb =$ the Sum of all the Series $\odot yf$, $\odot ed$, $\odot ab$, &c. which do conſtitute the Solidity of the Half Zone *am*AG.

And becauſe $\square AC - \square CB = \square ab$, therefore $\odot AC - \odot ab = \odot Cb$. Conſequently $\odot AC \times Cb : - \frac{\odot AC + \odot ab : \times Cb}{3} = 2\,\odot AC + \odot ab. \times \frac{1}{3}\,Cb$ will be the Solidity of the half Zone.

Put $D = AG = 2AC$, $x = am$, and $H = bB = 2Cb$.

Then $\odot AC. = 7854DD$. $\odot ab = .7854\ xx$. And if we turn the common Factor .7854 into a Diviſor, 1.27323, and then take the Triple of that Diviſor, *viz.* 3.8197, the Reſult of the precedent Work will produce this following Theorem.

$$\text{Theor.}\left\{\frac{2DD + xx}{3.8197} : \times H = \right\}\text{the middle Zone } amNK.$$

Which in Words is thus: To twice the Square of the Axis AG, add the Square of the Diameter of the Fruſtum's Baſe, (*am*) and divide the Sum by 3.8197, then multiply the Quotient by the Height or Thickneſs of the middle Zone, and the Product will be the Solidity of the middle Zone required.

This is ſo plain and eaſy it needs no *Example.*

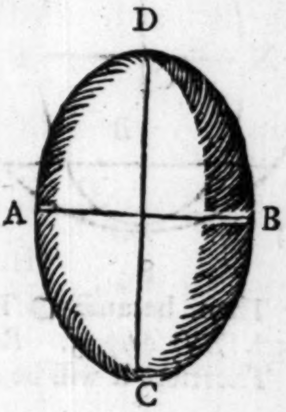
§ XII. *Of a* SPHEROID.

A Spheroid is a Solid reſembling an Egg. To find the ſolid Content thereof, this is

The RULE.

Multiply the Square of the Diameter of the greateſt Circle by the Length, and the Product mutiply again by .5236, this laſt Product will be the Solidity of the Spheroid.

Let AB, the Diameter of the greateſt Circle, be 33 Inches, and CD, the Length 55 Inches; the Solidity is required.



33
33

99
99

1089
55

5445
5445

59895

59895
.5236

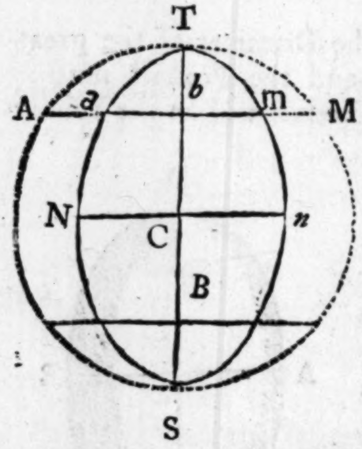
359370
179685
119790
299475

31361.0220 the Solidity

DEMON-

DEMONSTRATION.

Every Spheriod is equal to $\frac{2}{3}$ of a Cylinder, whoſe Baſe is equal to the greateſt Circle of the Spheriod, and its Height equal to the Length of the Spheroid.



Suppoſe the Figure NT*n*SN in the annexed Scheme, to repreſent a Spheroid, formed by the Rotation of the Semi-Ellipſis T N S, about its tranſverſe Axis TS.

Let $D=TS$, the Length of the Spheroid, and the Axis of its circumſcribing Sphere; and $d=Nn$ the Diameter of the greateſt Circle of the Spheroid.

Then, becauſe □ TC : NC : : □A*b* : □ *a b*, by Sect. XV. *Step* 3. *Page* 127.

Therefore it will be DD : dd : : □A*b* : □*ab*.

But the Sum of an infinite Series of ſuch Circles as ⊙A*b* (whoſe Diameters are Chords) do conſtitute the Solidity of the Sphere (by Sect. XI.)

And the Sum of an infinite Series of ſuch Circles as ⊙*ab*, (*viz.* whoſe Diameters are Ordinates of the Ellipſis) do conſtitute the Solidity of the Spheroid.

Therefore DD : dd : : 0.5236 DDD : 0.5236 Ddd = the Solidity of the Spheroid, (*Eucl.* 5. 12.)

But 0.5236 Ddd = $\frac{2}{3}$ of the Cylinder, whoſe Diameter is = d, and Height = D. by Sect. V.)

Now, from this Proportion between the Sphere and its inſcribed Spheroid, it will be very eaſy to deduce Theorems for finding the ſolid Content eithe of the Fruſtum or middle Zone of any Spheroid having the ſame Height of that with the Sphere; fo

As the Solidity of the whole Sphere : is to the Solidity of the whole Spheroid : : ſo is any Part of the Sphere, to the like Part of the Spheroid.

As for Inſtance, ſuppoſe it was required to find the middle Zone of any Spheroid.

Let $D=TS$ and $d=Nn$ as above, and $H=bB$. $x=AM$, and $c=am$.

Then, $\left\{\frac{2\,DD+xx}{3.8197}\times H\right.$ = the middle Zone of the Sphere

And $0.5236\,DDD : 0.5236\,ddD :: \frac{2DD+xx}{3.8197}\times H : \frac{2ddH}{3.8197}+\frac{xxddH}{3.8197\,DD}=$ the middle Zone of the Spheroid.

Again, $DD : dd :: xx : cc$. Therefore $\frac{xxdd}{DD}=cc$. Conſequently, $\frac{xxdd}{DD}\times\frac{H}{3.8197}=\frac{cc}{3.8197}\times H$. Which being taken inſtead of $\frac{xxddH}{3.8197DD}$, there will ariſe This following Theorem, $\left\{\frac{2dd+cc}{3.8197} : \times H\right.$ = the middle Zone of the Spheroid.

NOTE, That $3.8197=1.2732\times 3$. *See* p. 109.

---

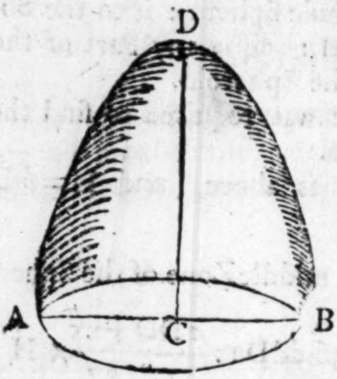
§ XII. *Of a Parabolick* CONOID.

A Parabolick Conoid is ſomething like an half Spheroid, having its Sides ſomewhat ſtraighter. It is generated by ſuppoſing a Semi-Parabola turned about its Axis. To find the ſolid Content thereof, this is

The RULE,

Multiply the Square of the Diameter of its Baſe by .7854, and multiply that Product by half the Height, that laſt Product ſhall be the ſolid Content.

Let



Let ABCD be a Parabolick Conoid, the Diameter of whoſe Baſe is 36 Inches and its Height CD 33 Inches; the Solidity is required.

36
36
216
108
1296

.7854
1296
47124
70686
15708
7854
1017.8784

1017.8784
33
30536352
30536352
2) 33589.9872
16794.9936

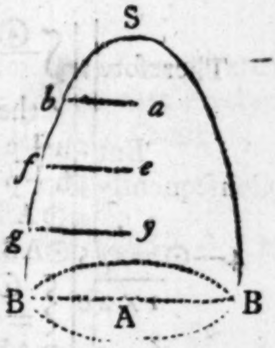
1728) 16794.99|36 (9.71 Feet, the Content.
15552
12429
12096
3339
1728
1611

DEMONSTRATION.

The Parabolick Conoid is conſtituted of an infinite Number of Circles, whoſe Diameters are the Ordinates of the Parabola. Now, according to the Property of every Parabola, it will be, SA : AB : : AB

$\frac{\square AB}{SA} = L$, the *Latus Rectum*,

Then



$$\text{Then}\begin{cases} Sa\times L = \square\, ba, \\ Se\times L = \square\, fe, \\ Sy\times L = \square\, gy, \ \&c. \end{cases}$$

Here $Sa\times L$. $Se\times L$, $Sy\times L$, &c. are a Series of Terms in Arithmetical Progreſſion.

Therefore $\square\, ba$, $\square\, fe$, $\square$ gy, &c. are alſo a Series of Terms in the ſame Progreſſion, beginning at the Point S, wherein $\square AB$ is the greateſt Term, and SA the Number of all the Terms. Therefore $\square AB\times\frac{1}{2}SA$ =the Sum of all the Series, (by *Lemma* II.)

Conſequently, $\odot AB\times\frac{1}{2}SA$ = the Sum of all the Series of $\odot\, ba$, $\odot\, fe$, $\odot\, gy$, &c. which do conſtitute the Solidity of the Conoid.

Put $D=2AB$, and $H=SA$.

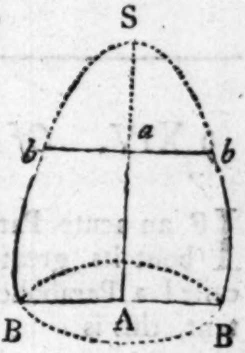
Then $.7854\, DD\times\frac{1}{2}H=.3927\,DDH$ will be the ſolid Content of the Conoid, which is juſt half the Cylinder whoſe Baſe is $=D$, and Height $=H$.

This being rightly underſtood, it will be eaſy to aiſe a Theorem for finding the lower Fruſtum of any arabolick Conoid.

For ſuppoſing $h=aA$, the Height of the Fruſtum, nd $p=Sa$, the Height of the Part *bSb* cut off; then $+p=SA$, the Height of the whole Conoid.

Conſequently $\dfrac{\odot AB\times h+\odot AB\times p}{2}$ = the Solidity of the whole onoid.

And $\dfrac{\odot\, ba\times p}{2}=$ the Solidity f the Part cut off.



Therefore	1	$\frac{\odot AB\times h + \odot AB\times p - \odot ba\times p}{2}$ the Solidity of the Fruſtum.
But	2	$h + p : \square AB :: p : \square ba.$
Conſequently	3	$h + p : \odot AB :: p : \odot ba.$
$3 \because$	4	$\odot AB\times p = \odot ba\times h + \odot ba\times p.$
$4 - \odot ba\times p$	5	$\odot AB\times p : - \odot ba\times p = \odot ba\times h.$
1×2	6	$\odot AB\times h : + \odot AB\times p : - \odot ba\times p = 2F.$
$6 - 5$	7	$\odot AB\times h = 2F : - \odot ba\times h.$
$7 + \odot ba\times h$	8	$\odot AB\times h : + \odot ba\times h = 2F.$
$8 \div 2$	9	$\frac{\odot AB + \odot ba}{2} : \times h = F$, the Fruſtum's Solidity.

Let $D = 2AB$, as before, and $d = 2ba$, the Diameter of the Part cut off; then we ſhall have this following Theorem.

$0.3927DD + 0.3927dd : \times h =$ the Solidity of the Fruſtum required. Which in Words is thus:

Multiply the Sum of the Squares of the greate and leſſer Diameters by .3927, and that Product b the Height of the Fruſtum, the laſt Product ſhall b the ſolid Content.

§ XIV. *Of a Parabolick* SPINDLE.

IF an acute Parabola be ſuppoſed to be moved bout its greateſt Ordinate, it will form a Soli called a Parabolick Spindle. To find the Solid Content, this is

Th

The RULE

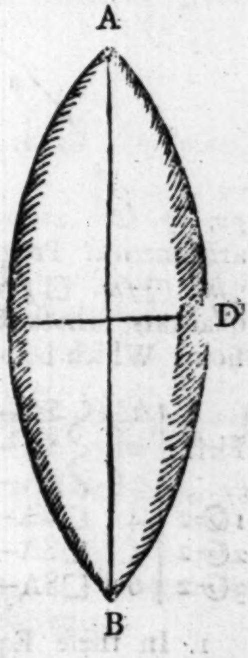
Multiply the Square of the Diameter of its greateſt Circle by .4188 being $\frac{8}{15}$ of .7854) and that Product by its Length, that laſt Product is the ſolid Content.

Let ABCD be a Parabolick Spindle, whoſe greateſt Diameter CD is 36 Inches, and its Length AB 99 Inches; the Solidity is required.

36=CD
36
216
108
1296 Square

.41888
1296
251328
376992
83776
41888
542.86848
99
488581632
488581632

1728)53743·97952(31·10184
1903
1759
3179
14515
6912



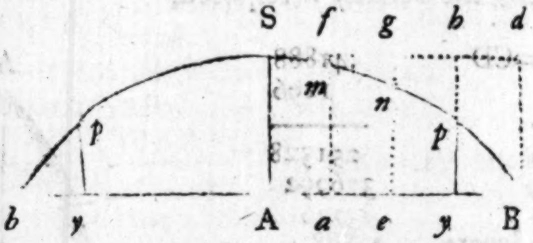
The ſolid Content is 31.10184 Feet.

DEMON-

DEMONSTRATION.

A Parabolick Spindle is conſtituted of an infinite Series of Circles, whoſe Diameters are all parallel to the Axis of a Parabola, as $\odot ma$, $\odot ne$, $\odot py$, &c.

Let us ſuppoſe the Line S*d* parallel to AB, *&c.* then it hath already been proved, that the Lines *fm*,



gn, *hp*, *&c.* are a Series of Squares whoſe Roots are in arithmetical Progreſſion; conſequently their Squares *viz.* $\square fm$ $\square gn$, $\square hp$, *&c.* will be a Series of Biquadrats, whoſe Roots will be in arithmetical Progreſſion: Which being premiſed, we may proceed thus;

Firſt,	1	$SA - fm = ma.$
	2	$SA - gn = ne.$
	3	$SA - hp = py.$
$1 \odot 2$	4	$\square SA - 2SA \times fm + \square fm = \square ma.$
$2 \odot 2$	5	$\square SA - 2SA \times gn + \square gn = \square ne.$
$3 \odot 2$	6	$\square SA - 2SA \times hp + \square hp = \square py$, *&c.*

1. In theſe Equations, the $\square SA$, $\square SA$, $\square SA$, being a Series of Equals, and AB the Number of all the Terms; therefore it will be $\square SA \times AB =$ the Sum of all the Series, by *Lemma* I.

2. Becauſe *fm*, *gn*, *hp*, &c. are a Series of Squares wherein SA is the greateſt Term, and AB the Number of all the Terms;

therefore $\frac{2SA \times SA \times AB}{3} = \frac{2 \square SA \times AB}{3}$ will be the Sum of all the Series, by *Lemma* III.

3. And the $\square fm$, $\square gn$, $\square hp$, &c. will be a Series of Terms in the Ratio of Biquadrats, as above, $\square SA$ being the greateſt Term, and AB the Number of all the Terms; therefore it will be $\frac{\square SA \times AB}{5}$ = the Sum of all the Series, by *Lemma* V.

Whence it follows, that $\square SA \times AB - \frac{2\square SA \times AB}{3} + \frac{\square SA \times AB}{5}$ = the Sum of all the Series of $\square ma$, $\square ne$, $\square py$, &c.

That is, $\frac{8\square SA \times AB}{15}$ = the Sum of all the Series, $\square ma$, $\square ne$, $\square py$, &c. Conſequently, $\frac{8\odot SA \times AB}{15}$ = the Sum of all the Series of Circles, $\odot ma$, $\odot ne$, $\odot py$, &c. which conſtitute the Solidity of half the Spindle, *viz.* of SAB.

Therefore putting D=2SA, and H=2AB, it will be 0.41888 DDH= the Solidity of the whole Parabolick Spindle *b*SB, being $\frac{8}{15}$ of 0.7854 DDH, the Solidity of its circumſcribing Cylinder.

From hence we may alſo raiſe a Theorem for finding the Fruſtum SA*py* of the laſt Figure.

For $\odot SA$ being the greateſt Term, $\odot py$ the leaſt Term, and A*y* the Number of all the Terms or Circles included between A and *y*.

Therefore, | 1 | $\left\{ \square SA - \frac{2SA\times bp}{3} + \frac{\square bp}{5} : \times Ay = z \right.$ the Sum of all the Series, $\square SA$, $\square ma$, $\square ne$, $\square py$.

1×3 | 2 | $3\square SA - 3SA\times bp + \frac{3\square bp}{5} : \times Ay = 3z$.

$2 \div Ay$ | 3 | $3\square SA - 2SA\times bp + \frac{3\square bp}{5} = \frac{3z}{Ay}$

But | 4 | $\square SA - 2SA\times bp = \square py - \square bp$, *per Step* 6.

$3 - 4$ | 5 | $2\square SA + \frac{3\square bp}{5} + \frac{3z}{Ay} - \square py + \square bp$.

$5 +$ &c. | 6 | $2\square SA + \square py - \frac{2}{5}\square bp = \frac{3z}{Ay}$

Conſeq. | 7 | $2\odot SA + \odot py - \frac{2}{5}\odot bp : \times \frac{1}{3}Ay = z$, the Sum of all the Series of $\odot SA$, $\odot ma$, $\odot ne$, $\odot py$, which do conſtitute the Solidity of the Fruſtum $SApy$.

Therefore putting $D = 2SA$, as before, $C = 2py$, $x = 2bp$, and $H = Ay$, it will be $1.5708\,DD + .7854\,CC - .31416\,xx : \times \frac{1}{3}H =$ the Fruſtum $SApy$: And if we make $L = 2H$, then $1.5708\,DD + .7854\,CC - .31416\,xx : \times \frac{1}{3}L =$ the Double of that Fruſtum, being the middle Zone: Which in Words is thus:

Multiply the Square of the greateſt Diameter by 1.5708, and multiply the Square of the leſſer Diameter by .7854, and multiply the Square of the Difference of the Diameters by .31416; from the Sum of the two former Products ſubtract the latter Product, and multiply the Remainder by one third Part of the Length, and that Product will be the Solidity of the middle Zone required.

CHAP. III.

Of Meaſuring of the Works of the ſeveral Artificers relating to Building; and what Methods and Cuſtoms are obſerved therein.

§ I. *Of* CARPENTERS WORK.

THE Carpenters Works which are meaſurable, are, Flooring, Partitioning and Roofing; all which are meaſured by the Square of 10 Feet long and 10 Feet broad; ſo that one Square contains 100 ſquare Feet.

1. *Of* Flooring.

If a Floor be 57 Feet 3 Inches long, and 28 Feet 6 Inches broad, how many Squares of Flooring are there in the Room.

Multiply 57 Feet 3 Inches by 28 Feet 6 Inches, and the Product is 1631 Feet, *&c.* which divide by 100; (this is done by cutting off from the Product two Figures towards the Right Hand with a Daſh of the Pen) the remaining Figures are the Quotient, and the Figures cut off are Feet Thus, 1631 divided by 100, by cutting off 31 from the Right Hand thereof, the Quotient is 16 Squares and 31 cut off, is 31 Feet.

See the Work, both by Decimals, and alſo by Feet and Inches.

	F.	I.	
57.25	57	3	
28.5	28	6	
28625	456		
45800	114		
11450	28	7	6
	7	0	0
16|31.625	16|31	7	6

Facit, 16 Squares and 31 Feet.

NOTE, That .5 is the Decimal for a Half of any Thing, .25 is the Decimal for a Quarter, and .125 is the Decimal for half a Quarter; ſo, in the laſt *Example,* .25 is the Decimal of 3 Inches, becauſe 3 Inches is a Quarter of a Foot; and 5 is the Decimal of 6 Inches, becauſe 6 Inches is half a Foot.

Example 2. Let a Floor be 53 Feet 6 Inches long, and 47 Feet 9 Inches broad, how many Squares are contained in that Floor?

	F.	I.	
47.75	53	6	
53.5	47	9	
23875	371		
14325	212		
23875	26	9	
	13	4	6
25|54.625	23	6	
	25|54	7	6

Facit, 25 Squares and 54 Feet.

By Scale *and* Compaſſes.

In the firſt *Example* extend the Compaſſes from 1 to 28.5, that Extent will reach from 57.25 to 16 Squares and near a third Part.

In the ſecond *Example,* extend the Compaſſes from 1 to 47.75, that Extent will reach from 53.5 to 25 Squares and above an half.

2. *Of* Partitioning.

Example 1. If a Partition between Rooms be in Length 82 Feet 6 Inches, and in Height 12 Feet 3 Inches, how many Squares are contained therein?

The Length and Breadth being multiplied together, the Product is 1010.625; which divide by 100, (as before is ſhewed) and the Anſwer is 10 Squares 10 Feet; the Inches or Parts in theſe Caſes are of no Value.

```
     12.25           F.  I.
     82.5            82  6
   ---------         12  3
     6125          ----------
    2450           990  0
   9800             20  7  6
   ---------       ----------
  10|10.625       10|10  7  6
```

Facit, 10 Squares 10 Feet.

Example 2. If a Partition between Rooms be in Length 91 Feet 9 Inches, and in Breadth 11 Feet 3 Inches, how many Squares are contained therein?

The Length and Breadth being multiplied together, the Product is 1032 Feet; which divided by 100, the Anſwer will be 10 Squares and 32 Feet.

```
      91.75           F.  I.
      11.25           91  9
    ---------         11  3
     45875          ----------
    18350          1009  3
    9175             22 11  3
   9175             ----------
   ---------       10|32  2  3
  10|32.1875
```

3. *Of*

3. *Of* Roofing.

It is a Rule amongſt Workmen, that the Flat of any Houſe and half the Flat thereof, taken within the Walls, is equal to the Meaſure of the Roof of the ſame Houſe; but this is when the Roof is true pitched. For if the Roof be more flat or ſteep than the true Pitch, it will meaſure to more or leſs accordingly.

Example 1. If a Houſe within the Walls be 44 Feet 6 Inches long, and 18 Feet 3 Inches broad, how many Squares of Roofing will cover that Houſe?

Multiply the Length and Breadth together, and the Product is 812 Feet, the Flat; the Half thereof is 406 Feet which added to the Flat, the Sum is 1218 Feet; which divided by 100, the Anſwer is 12 Squares and 18 Feet.

			F.	I.	
	18.25		44	6	
	44.5		18	3	
	9125		352		
	7300		44		
	7300		11	1	6
			9	0	0
Flat.	812.125				
Half.	406	The Flat.	812	1	6
		The Half	406		
	12|18				
		Sum.	12|18		

Facit, 12 Squares 18 Feet.

By Scale *and* Compaſſes.

In the firſt *Example* of Partitioning, extend the Compaſſes from 1 to 12.25, that Extent will reach from 82.5 to 10 Squares and one Tenth.

In the ſecond *Example*, extend the Compaſſes from 1 to 11.25, that Extent will reach from 91.75 to 10 Squares, and a little leſs than a third Part.

In the *Example* of Roofing, extend the Compaſſes from 1 to 18.25, that Extent will reach from 44.5 to 812, the Flat; to which add the Half thereof, and the Sum is 12.18, which is 12 Squares 18 Feet, as above.

There are other Works about a Building done by the Carpenter, that are meaſured by the Foot, running Meaſure, that is, by the Number of Feet in Length only; as Cornices, Doors and Caſes, Window Frames, Guttering, Lintels, Sommers, Skirt-boards, *&c.*

NOTE 1. In meaſuring of Flooring, after you have meaſured the whole Floor, you muſt deduct out of it the Well holes for the Stairs and Chimneys; and in Partitioning, for the Doors, Windows, *&c.* except (by Agreement) they are to be included.

NOTE 2. In meaſuring of Roofing, ſeldom any Deductions are made for the Holes for the Chimney Shafts, the Vacancies for Lutheren-Lights and Sky-Lights; for they are more Trouble to the Workman, than the Stuff which would cover them is worth.

---

§ II. *Of* BRICKLAYERS WORK.

THE principal is Tiling, Walling, and Chimney-Work.

1. *Of* Tiling.

Tiling is meaſured by the Square of 10 Feet, as Flooring, Partitioning and Roofing were in the Carpenter's Work; ſo that between the Roofing and Tiling, the Difference will not be much, yet the Tiling will be the moſt; for the Bricklayers ſometimes

times will require to have double Meaſure for Hips and Vallies. When Gutters are allowed double Meaſure, the Way is to meaſure the Length along the Ridge Tile, and by that Means the Meaſure of the Gutters becomes double; it is uſual alſo to allow double Meaſure at the Eaves, ſo much as the Projecture is over the Plate, which is commonly about 18 or 20 Inches.

Example 1. There is a Roof covered with Tiles, whoſe Depth on both Sides (with the uſual Allowance at the Eaves) is 37 Feet 3 Inches, and the Length 45 Feet; I demand how many Squares of Tiling are contained therein?

F. I.
37 3 — 37.25
45 0 — 45

185 — 18625
148 — 14900
11 3 — 16|76.25

16|76 3

Anſwer. 16 Squares 76 Feet.

Example 2. There is a Roof covered with Tiles, whoſe Depth on both Sides (with the Allowance at the Eaves) is 35 Feet 9 Inches, and the Length 43 Feet 6 Inches; I demand how many Squares of Tiling are in the Roof?

```
  F.  I.
  43  6
  35  9
 --------
  215
 129
  21  9
  10 10  6
  17  6
 --------
15|55  1  6
```

```
   35.75
    43.5
 --------
   17875
  10725
 14300
 --------
15|55.125
```

Here the Length and Depth being multiplied together, the Product is 1555 Feet; which divided by 100, (as before is taught) the Anſwer is 15 Squares and 55 Feet.

By Scale *and* Compaſſes.

In the firſt *Example*, extend the Compaſſes from 1 to 37.25, that Extent will reach from 45 to 16 Squares, and a little above three Quarters of a Square.

In the ſecond *Example*, extend the Compaſſes from 1 to 35.75, that Extent will reach from 43.5 to 15 Squares and 55 Feet, that is, a little above a half Square.

2. *Of* Walling.

Bricklayers commonly meaſure their Work by the Rod-ſquare of 16 Feet and a half; ſo that one Rod in Length and one in Breadth, contain 272.25 ſquare Feet; for 16.5 multiplied in itſelf, produces 272.25 ſquare Feet: But in ſome Places the Cuſtom is to allow 18 Feet to the Rod, that is, 324 ſquare Feet; and in ſome Places the uſual Way is to meaſure by the Rood of 21 Feet long and 3 Feet high, that is, 63 ſquare Feet; and here they never regard the Thickneſs of the Wall, but the uſual Way is to moderate the Price according to the Thickneſs.

But in *Ireland* thy meaſure by a Perch of 21 Feet in Length, and 1 Foot in Height, without any regard to Thickneſs in the Meaſure, eſpecially in Stone-work.

When you meaſure a Piece of Brick-work, the firſt Thing is to enquire by which of theſe Ways it muſt be meaſured, then having multiplied the Length and Breadth in Feet together, divide the Product by the Proper Diviſor, either for Rods, or Roods, and the Quotient is ſquare Rods or ſquare Roods accordingly.

But in *England*, commonly Brick Walls, that are meaſured by the Rod, are to be reduced to a Standard Thickneſs, *viz.* of a Brick and a half thick, (if it be not agreed on to the contrary) and to reduce a Wall to Standard Thickneſs, this is

The RULE.

Multiply the Number of ſuperficial Feet that are found to be contained in any Wall, by the Number of half Bricks which that Wall is in Thickneſs, one third Part of that Product ſhall be the Content thereof in Feet, reduced to the Standard Thickneſs of one Brick and a half.

Example 1. If a Wall be 72 Feet, 6 Inches long, and 19 Feet 3 Inches high, and 5 Bricks and a half thick, how many Rods of Brick-work are contained therein when reduced to the Standard?

9:25

19.25 Height
72.5 Length.

9625
3850
13475

1395.625
11

3) 15351.875

272.25) 5117.291 (18 Rods.

239479

68.06) 21679 (3 Quarters of a Rod.

12.61

Anſwer, 18 Rods, 3 Quarters, 12 Feet.

F.	I.	
72	6	
19	3	
648		
72		
18	1	6
9	6	0
1395	7	6
11		

3) 15345

272) 5115 (18 Rods.

2395

68) 219 (3 Quarters of a Rod.

15

NOTE, That 68.06 is one fourth Part of 272.25.

NOTE alſo, That in reducing of Feet into Rods, they uſually reject the odd Parts, and divide only by 272, as is done in that ſecond Way of the laſt *Example*; ſo the Anſwer, by that ſecond Way, is 18 Rods, 3 Quarters, and 15 Feet, more by about $2\frac{1}{2}$ Feet than by the firſt Way, where it is done decimally; a Thing very inſignificant.

Example 2. If a Wall be 245 Feet 9 Inches long, and 16 Feet 6 Inches high, and 2 Bricks and a half thick; I demand how many Rods of Brick-work are contained therein, when reduced to Standard Thickneſs?

```
     245.75
      16.5
     ------
     122875
    147450
    24575
    --------
    4054.875
       5
    -----
 3) 20270
    -----
272) 6756 (24 Rods.
    ----
    1316
    ----
 68) 228 (3 Quarters of a Rod.
    ----
     24
```

Anſwer. 24 Roods, 3 Quarters, 24 Feet.

F. I.
245 9
16 6

1470
245
122 10 6
12 0 0

4054 10 6 *Anſwer* in Feet.

Before I ſhew how to work the two laſt *Examples* by Scale and Compaſſes, I will ſhew how to find proper Diviſors to facilitate the Operation, becauſe it would be too intricate and tedious to perform by Scale and Compaſſes, according to the Rule above taught.

To find the proper Diviſors.

Divide 3 (the Number of half Bricks in 1 and ½) by the Number of half Bricks in the Thickneſs, the Quotient will be a Diviſor which will give the Anſwer in Feet. But if you would have a Diviſor to bring the Anſwer into Rods at once, then multiply 272.25 by the Diviſor found for Feet, and the Product will be a Diviſor which will give the Anſwer in Rods.

Example. Let it be required to find a Diviſor proper to reduce a Wall of three Bricks thick.

Divide 3 by 6, (the half Bricks in the Thickneſs) and the Quotient is 5, which is a Diviſor that will give the Anſwer in Feet. Then multiply 272.25 by .5, and the Product is 136.125, the Diviſor which will give the Anſwer in Rods; that is, as 136.125 is to the Length of the Wall, ſo is the Height to the Content in Rods. Or, as .5 is to the Length, ſo is the Height to the Content in Feet.

After the ſame Manner you may find Diviſors for any other Thickneſs, which you will find to be as expreſſed in the following little Table.

The Thickneſs of the Wall.	Diviſors for the Anſwer in Feet	Diviſors for bringing the Anſwer in Rods.
1 Brick thick	1.5	408.375
1 and half Brick thick	1.	272.25
2 Bricks thick	.75	204.1875
2 and half Bricks thick	.6	163.35
3 Bricks thick	.5	136.125
3 and half Bricks thick	.4285	116.658
4 Bricks thick	.375	102.0937

Let the ſecond *Example*, aforegoing, be wrought by Scale and Compaſſes, where the Length is 245.75 the Height 16.5, and the Thickneſs is 2 and half Bricks.

Extend the Compaſſes from 163.35 (the tabular Number againſt 2 and half Bricks) to 245.75, that Extent will reach from 16.5 to 24 Rods and 8 Tenths.

Again, if the Length be 75 Feet 6 Inches, and the Height 18 Feet 9 Inches, at 3 and half Bricks thick, how many Rods are contained therein?

Extend the Compaſſes from 116.678 (the tabular Number) to 18.75, that Extent will reach from 75.5 to 12.13; that is, 12 Rod, and a little above half a Quarter.

In *Britain* it will be very proper and commodious, for ſuch as have frequent Occaſion to meaſure Brick-work, to have in the Line of Numbers little Braſs Center-pins at each of the Numbers in the third Column of the above little Table, with a Figure to denote the Thickneſs of the Wall.

If a Wall be 104 Feet 9 Inches long, and 17 Feet 3 Inches high, how many Rods are contained therein?

```
                                    F.   I.
        104.75                     104   9
         17.25                      17   3
      -----------                 ------------
         52375                     728
        20950                     104
        73325                       26   2   3
       10475                        12   9   0
      -----------                 ------------
   63) 1806.9375 (28              1806  11   3
       126
      -----     Anſwer.   28 Rods, 42 Feet.
        546
        504
       -----
         42
```

NOTE, That ſuch as dig Cellars do many times do them by the Floor, 18 Feet ſquare, and a Foot deep, being a Floor of Earth, that is, 324 ſolid Feet.

3. *Of* Chimnies.

If you are to meaſure a Chimney ſtanding alone by itſelf, without any Party-wall being adjoined, then girt it about for the Length, and the Height of the Story is the Breadth; the Thickneſs muſt be the ſame as the Jaums are of, provided that the Chimney be wrought upright from the Mantle-tree to the Cieling, not deducting any Thing for the Vacancy between the Floor (or Hearth) and the Mantle tree, becauſe of the Gatherings of the Breaſt and Wings, to make room for the Hearth in the next Story.

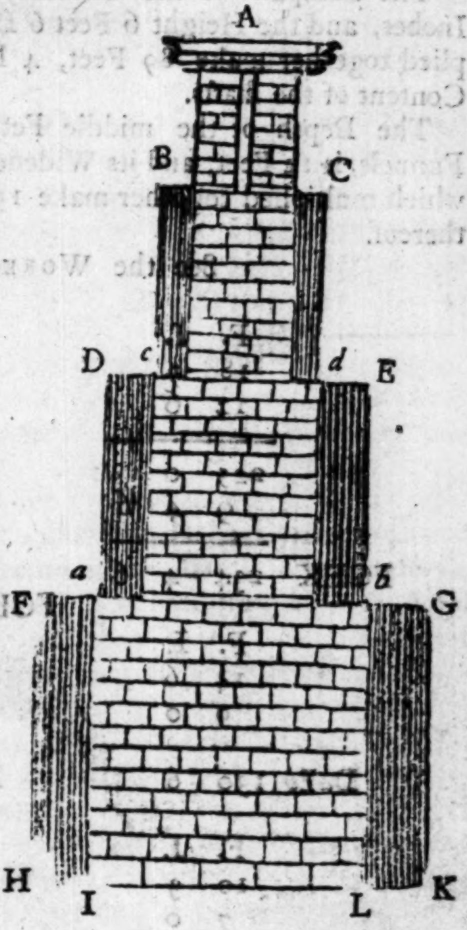
If the Chimney-back be a Party-wall, and the Wall be meaſured by itſelf, then you muſt meaſure the Depth of the two Jaums, and the Length of the Breaſt for a Length, and the Height of the Story the Breadth, at the ſame Thickneſs your Jaums were of.

When you meaſure Chimney-ſhafts, girt them with a Line round about the leaſt Place of them, for the Length, and the Height ſhall be your Breadth: And if they be four Inch Work, then you muſt ſet down your Thickneſs at one Brick-work; but if they be wrought 9 Inches thick, (as ſometimes they are, when they ſtand high and alone above the Roof) then you muſt account your Thickneſs one and half Brick, in conſideration of Wyths and Pargetting, and Trouble in Scaffolding.

It is cuſtomary in moſt Places to allow double Meaſure for Chimnies.

Example. Suppoſe this Figure, ABCDEFGHK, to be a Chimney that hath a double Funnel towards the Top, and a double Shaft, and is to be meaſured according to double Meaſure.

Firſt, I begin with the Breaſt-Wall IL, and the two Angles LK and HI, which together are 18 Feet 9 Inches; then take the Height of the Square HF, 12 Feet 6 Inches, which multiply'd together, produce 234 Feet, 4 Inches, 6 Parts, for the Contents of the Figure FGHK.



For the Square D*a*E*b*, the Length of the Breaſt Wall and two Angles, is 14 Feet 6 Inches, and the Height D*a* 9 Feet, which multiplied together make 130 Feet 6 Inches, for the Content of the Square D*a*E*b*.

Then the Height of the next Square 7 Feet, and the Length of the Breaſt-Wall and two Angles is 10 Feet, 3 Inches, which multiplied together produce 71 Feet 9 Inches, for the Content of the Square B*c*C*d*.

The Compaſs of the Chimney-ſhafts is 13 Feet 9 Inches, and the Height 6 Feet 6 Inches, which multiplied together make 89 Feet, 4 Inches, 6 Parts, the Content of the ſhafts.

The Depth of the middle Fetter, that parts the Funnels, is 12 Feet, and its Wideneſs 1 Foot 3 Inches, which multiplied together make 15 Feet, the Content thereof.

See the WORK.

	F.	I.			
	18	9			18.75
	12	6			12.5
	225	0			9375
	9	4	6		3750
					1875
FGHK	234	4	6	FGHK	234.375
	F.	I.			
	14	6			14.5
	9	0			9
D*a*E*b*	130	6		D*a*E*b*	130.5
	F.	I.			
	10	3			10.25
	7	0			7
B*c*C*d*	71	9		B*c*C*d*	71.75
	F.	I.			
	13	9			13.75
	6	6			6.5
	82	6			6875
	6	10	6		8250
The Shaft	89	4	6	The Shaft	89.375

F.	I.
1	3
12	0
The Fetter 15	0

272) 1082 (3 Rods.

68) 266 (3 Quarters.

Rem. 62 Feet.

1.25
12

The Fetter 15.00

	F.	I.	P.
FGHK	234	4	6
D*a*E*b*	130	6	0
B*c*C*d*	71	9	0
The Shaft	89	4	6
The Fetter	15	0	0
The Sum	541	0	0
The Double	1082	0	0

Having added the five Products together, and doubled the Sum, that double Sum is the Content of the Chimney in Feet, according to double or cuſtomary Meaſure; which Feet muſt be reduced to Rods, as was ſhewed before.

So the Feet in the foregoing *Example* being reduced to Rods, (the Thickneſs being ſuppoſed 1 and half Bricks) it makes 3 Rods, 3 Quarters, and 62 Feet, that is, 4 Rods, wanting 6 Feet.

This is all the Meaſure that can be allowed, when the Chimney ſtands in a Gavel, or Side-Wall; in which Caſe the Back of the Chimney (here not meaſured) is accounted as Part of the Gavel; but if the Chimnies ſtand by themſelves, as all Stacks of Chimnies in great Buildings do, which, in ſuch Caſe, is all Chimney-Work, and therefore ought to be meaſured double on all Sides.

§ III. *Of* PLAISTERERS Work.

THE Plaiſterers Works are principally of two Kinds, namely, 1. Works lath'd or plaiſter'd, which they call Cieling. 2. Works render'd, which is of two Kinds, *viz.* upon Brick Walls, or between Quarters in the Partitions between Rooms; all which are meaſured by the Yard ſquare, or Square of 3 Feet, which is 9 Feet.

1. *Of* Cieling.

If a Cieling be 59 Feet 9 Inches long, and 24 Fee 6 Inches broad, how many Yards doth that Cieling contain?

Multiply 59 Feet 9 Inches by 24 Feet 6 Inches, and the Product is 1463 Feet 10 Inches, 6 Parts which divided by 9, the Quotient is 162 Yards, Feet.

F.	I.		59.75
59	9		24.5
24	6		
			29875
236			23900
118			11950
29	10	6	
18	0	0	9)1463.875
1463	10	6	*Anſwer* 162.65

By Scale *and* Compaſſes.

Extend the Compaſſes from 9 to 59 Feet 9 Inch that Extent will reach from 24 Feet 6 Inches to 16 Yards.

2. *Of* Rendering.

Example. If the Partitions between Rooms 141 Feet 6 Inches about, and 11 Feet 3 Inches hi how many Yards are in thoſe Partitions?

Multiply 141 Feet 6 Inches by 11 Feet 3 Inch and the Product is 1591 Feet, 10 Inches, 6 Pa which divided by 9, gives 176 Yards 7 Feet, Anſwer.

F.	I.		
141	6		141.5
11	3		11.25
1556	6		7075
35	4	6	2830
			1415
9)1591	10	6	1415
F.	Y.		9)1591.875
Anſw. 176	7		
			176.87

Anſwer. 176.87 Yards.

Extend the Compaſſes from 9 to 141.5 that Extent will reach from 11.25 to 176.87 Yards.

NOTE 1. If there be any Doors, Windows, or the like, in your Partitioning, you muſt make Deductions for them.

NOTE 2. When you meaſure Rendering upon Brick-Walls, you are to make no Deductions; but when you meaſure Rendering between Quarters, you may very well deduct one fifth Part, for the Quarters, Braces, and Interſtices.

NOTE 3. That Whiting and Colouring are both meaſured by the Yard, as Cieling and Rendering were; and as in Rendering between Quarters you deduct one fifth Part, ſo in Whiting and Colouring you muſt add one fourth or one fifth Part, at leaſt.

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§ IV. *Of* JOYNERS Work.

JOYNERS do meaſure their Work by the Yard ſquare; but in taking their Dimenſions they differ from ſome others; for they have a Cuſtom, and ſay, *We ought to meaſure where our Plane touches*;

wherefore, in taking the Height of any Room, where there is a Cornice about, and ſwelling Pannels and Mouldings, they, with a String, begin at the Top, and girt over all the Mouldings, which will make the Room to meaſure much higher than it is: Then, for meaſuring about the Room, they only take it as it is upon the Floor.

Example 1. If a Room of Wainſcot (being girt downwards over the Mouldings) be 15 Feet 9 Inches high, and 126 Feet 3 Inches in Compaſs, how many Yards doth that Room contain?

Multiply the Compaſs by the Height, and the Product is 19.88 Feet, 5 Inches, 3 Parts; which divided by 9, gives 220 Yards and 8 Feet, the Anſwer.

	F.	I.	
	126	3	
	15	9	
	630		
	126		
	63	1	6
	31	6	9
	3	9	0
	9)1988	5	3
Anſwer,	220	8	

126.25	
15.75	
63125	
88375	
63125	
12625	
9)1988.4375	
220	8

Facit 220 Yards, 8 Feet.

Example 2. If a Room of Wainſcot be 16 Feet, 3 Inches high (being girt over the Mouldings) and the Compaſs of the Room 137 Feet 6 Inches, how many Yards are contained therein?

Multiply 137 Feet 6 Inches by 16 Feet 3 Inches, and the Product is 2234 Feet, 4 Inches, 6 Parts; which divided by 9, the Quotient is 248 Yards and 2 Feet.

F.	I.	
137	6	
16	3	
830		
137		
34	4	6
9)2234	4	6
248	2	0

137.5
16.25
6875
2750
8250
1375
9)2234.375
248 2

Facit 248 Yards, 2 Feet.

By Scale *and* Compaſſes.

For the firſt *Example*, extend the Compaſſes from 9 to 126.25, that Extent will reach from 15.75 to 220.8 Yards.

For the ſecond *Example*, extend the Compaſſes from 9 to 137.5, that Extent will reach from 16.25 to 248 Yards, and about a Quarter.

In Joyners Work there is another Thing to be obſerved, that is, in the meaſuring of Doors, Window-ſhutters, and all ſuch Work as is wrought on both Sides, they are paid for Work and half Work; ſo that in meaſuring all ſuch Work, you muſt firſt find the Content, as before, and take half that Content and add to it; ſo ſhall the Sum be the Content at Work and half.

Example. If the Window-ſhutters about a Room be 69 Feet 9 Inches broad, and 6 Feet 3 Inches high, how many Yards are contained therein at Work and half?

Multiply 69 Feet 9 Inches by 6 Feet 3 Inches, and the Product is 435 Feet, 11 Inches, 3 Parts; the half whereof is 217 Feet, 11 Inches, 7 Parts; which added together, the Sum is 653 Feet, 10 Inches, 10 Parts: which divided by 9, the Quotient is 72 Yards, 5 Feet, the Content at Work and half.

F.	I.		69.75
69	9		6.25
6	2		
			34875
418	6		13950
17	5	3	41850
435	11	3	435.9375
217	11	7	217.9687
9) 653	10	10	653.9062
72	5		*Facit* 72 Yards 5 Feet.

By Scale *and* Compaſſes.

Extend the Compaſſes from 9 to 69.75, that Extent will reach from 6.25 to 48.4 Yards; the Half whereof is 24.2 which added together make 72.6 Yards, the Content at Work and half.

NOTE, That you muſt make Deductions for all Window-Lights; but you muſt meaſure the Window-Boards, Sopheta-Boards and Cheeks by themſelves.

§ V. *Of* PAINTERS Work.

THE taking the Dimenſions of Painters Work is the ſame as that of Joyners, by girting over the Mouldings and Swelling Pannels, in taking the Height; and it is but Reaſon that they ſhould be paid for that on which their Time and Colour are both expended. The Dimenſions thus taken, the caſting up, and reducing Feet into Yards, is altogether the ſame as the Joyners Work; but the Painter never requires Work and half, but reckons his Work, once, twice, or thrice coloured over. Only take Notice, that Window-Lights, Window-Bars, Caſements, and ſuch like Things, they do at ſo much *per* Piece.

Example. If a Room be painted, whoſe Height (being girt over the Mouldings) is 16 Feet 6 Inches, and the Compaſs of the Room 97 Feet 9 Inches; how many Yards are in that Room?

Multiply 97 Feet 9 Inches by 16 Feet 6 Inches, and the Product is 1612 Feet, 10 Inches, 6 Parts; which being divided by 9, the Quotient is 179 Yards and 1 Foot.

F.	I.		
97	9		97.75
16	6		16.5
584			48875
98			58650
48	10	6	9775
9)1612	10	6	9)1612.875(179.1
179	1		*Facit* 179 Yards, 1 Foot.

By Scale *and* Compaſſes.

Extend the Compaſſes from 9 to 16.5, that Extent will reach from 97.75 to 179.2 Yards.

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§ VI. *Of* GLAZIERS Work.

GLAZIERS do meaſure their Work by the Foot ſquare, ſo that the Length and Breadth of a Pane of Glaſs in Feet, being multiplied into each other, produceth the Content.

NOTE, That Glaziers do uſually take their Dimenſions to a Quarter of an Inch; and in multiplying Feet, Inches, and Parts, the Inch is divided into 12 Parts, as the Foot is, and each Part ſubdivided into 12, *&c.*

Example 1. If a Pane of Glaſs be 4 Feet, 8 Inches and 3 Quarters long, and 1 Foot 4 Inches, 1 Quarter broad, how many Feet of Glaſs in that Pane?

The Decimal of $\left\{\begin{matrix}8\text{ Inches }\frac{3}{4}\\ 4\text{ Inches }\frac{1}{4}\end{matrix}\right\}$ is $\left\{\begin{matrix}.729\\ .354\end{matrix}\right.$

F.	I.	P.			
4	8	9			4.729
1	4	3			1.354
4	8	9			18916
1	6	11	0		23645
	1	2	2	3	14187
					4729
6	4	10	2	3	
					6.403066

Anſwer, 6 Feet 4 Inches.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 1.354, that Extent will reach from 4.729 to 6.4 Feet the Content.

Example 2. If there be eight Panes of Glaſs, each 4 Feet, 7 Inches, 3 Quarters long, and 1 Foot, 5 Inches, 1 Quarter broad; how many Feet of Glaſs is contained in the ſaid eight Panes?

The Decimal of $\left\{\begin{matrix}7\text{ Inches }\frac{3}{4}\\ 5\text{ Inches }\frac{1}{4}\end{matrix}\right\}$ is $\left\{\begin{matrix}.646\\ .437\end{matrix}\right.$

F. I.

F.	I.	P.			4.646
4	7	9			1.437
1	5	3			
					32522
4	7	9			13938
1	11	2	9		18584
	1	1	11	3	4646
6	8	1	8	3	6.676302
				8	8
53	5	1	6	0	53.410416

Facit 53 Feet, 5 Inches.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 1.437, that Extent will reach from 4.646 to 6.676; then extend the Compaſſes from 1 to 8, that Extent will reach from 6.676 to 53.4 the Content.

Example 3. If there be 16 Panes of Glaſs, each 4 Feet, 5 Inches and a half long, and 1 Foot, 4 Inches 3 Quarters broad, how many Feet of Glaſs are contained therein?

F.	I.	P.			4.458
4	5	6			1.395
1	4	9			
					22290
4	5	6			40122
1	5	10	0		13374
	3	4	1	6	4458
6	2	8	1	6	6.218910
				4	4
24	10	8	6	0	24.87564
				4	4
99	6	10	0	0	99.50256

Facit 99 Feet, 6 Inches.

NOTE, that inſtead of multiplying by 16, I have multiplied by 4 twice, becauſe 4 times 4 is 16.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 1.395, that Extent will reach from 4.458 to 6.219; then extend the Compaſſes from 1 to 16, that Extent will reach from 6.219 to 99.5 Feet, the Content.

NOTE, That when Windows have half Rounds at the Top, they meaſure them at the full Height, as if they were ſquare. Alſo round or oval Windows are meaſured at the full Length and Breadth of their Diameters. Likewiſe Crocket Windows in Stonework are all meaſured by their full Squares; and there is Reaſon for ſo doing; for the Trouble in taking their Dimenſions to work by, the Waſte of Glaſs in working, and the Time expended in ſetting up, is far more than the Glaſs can be valued at.

§ VII. *Of* MASONS Work.

MASONS do meaſure their Work ſometimes by the Foot ſolid, ſometimes by the Foot ſuperficial: and in ſome Places they meaſure their Walling by the Rood, that is 21 Feet long and 3 Feet high, which is 63 ſquare Feet.

Examples of each are as follow.

Example 1. If a Wall be 97 Feet 5 Inches long, 18 Feet 3 Inches high, and 2 Feet 3 Inches thick, how many ſolid Feet are contained in that Wall?

F. I.

F.	I.			
97	5			97.417
18	3			18.25
776				487085
97				194834
24	4	3		779336
6	0	0		97417
1	6	0		
				1777.86225
1777	10	3		2.25
2	3			
				888930125
3555	8	6		355572050
444	5	6	9	355572050
4000	2	0	9	4000.1855625

Multiply the Length, Height and Thickneſs together, and the laſt Product is 4000 Feet 2 Inches, the ſolid Feet contained in the Wall.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 18.25, that Extent will reach from 97.417 to 1777.86; then extend from 1 to 1777.86, that Extent will reach from 2.25 to 4000.18, the ſolid Content.

Example 2. If a Wall be 107 Feet 9 Inches long, and 20 Feet 6 Inches high, how many Feet ſuperficial are contain'd therein?

F.	I.		
107	9		107.75
20	6		20.5
2155	0		53875
53	10	6	215500
2208	10	6	2208.875

Facit 2208 Feet, 10 Inches.

By

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 107.75, that Extent will reach from 20.5 to 2208.875, the ſuperficial Feet.

Example 3. If a Wall be 112 Feet 3 Inches long, and 16 Feet 6 Inches high, how many Roods are contained therein?

F.	I.		
112	3		112.25
16	6		16.5
676	0		56125
112			67350
56	1	6	12225
1852	1	6	63)1852.125(29
			592
			25

Facit 29 Roods 25 Feet.

By Scale *and* Compaſſes.

Extend the Compaſſes from 63 to 16.5, that Extent will reach from 112.25 to 29.4 Roods, the Content.

CHAP. IV.

The Meaſuring of BOARD *and* TIMBER.

---

§ I. *Of* BOARD-MEASURE.

TO meaſure a Board, is no other but to meaſure a long Square.

Example 1. If a Board be 16 Inches broad, and 13 Feet long, how many Feet are contained therein?

Multiply 16 by 13, and the Product is 208; which divided by 12, gives 17 Feet, and 4 remains, which is a third Part of a Foot.

Or thus; Multiply 156 the Length in Inches) by 16, and the Product is 2496? which divided by 144, the Quotient is 17 Feet, and 48 remains, which is the third Part of 144; the ſame as before.

As 12 : 13 :: 16
$$\begin{array}{r} 13 \\ \hline 48 \\ 16 \\ \hline 12)208 \\ \hline 17\frac{4}{12} \end{array}$$

Or, As 144 : 156 :: 16
$$\begin{array}{r} 16 \\ \hline 936 \\ 156 \\ \hline 144)2496(17\frac{48}{144} = \frac{1}{3} \\ \hline 1056 \\ \hline 48 \end{array}$$

By Scale *and* Compaſſes.

Extend the Compaſſes from 12 to 13, that Extent will reach from 16 to 17 $\frac{1}{3}$ Feet the Content.

Or, Extend from 144 to 156, (the Length in Inches) that Extent will reach from 16 to 17 $\frac{1}{3}$ Feet, the Content.

Example 2. If a Board be 19 Inches broad, how many Inches in Length will make a Foot?

Divide 144 by 19, and the Quotient is 7.58 very near; and ſo many Inches in Length, if a Board be 19 Inches broad, will make a Foot.

Inch. Inch. Inch. Inch.
As 19 : 144 :: 1 : 758 *ferè*,

Extend the Compaſſes from 19 to 144, that Extent will reach from 1 to 7.58, that is, 7 Inches and ſomething

thing more than a half. So if a Board be 19 Inches broad, if you take 7 Inches and a little more than a half with your Compaſſes from a Scale of Inches, and run that Extent along the Board, from End to End, you may find how many Feet that Board contains, or you may cut off from that Board any Number of Feet deſir'd.

For this Purpoſe there is a Line upon moſt ordinary Joint-Rules, with a little Table placed upon the End, of all ſuch Numbers as exceed the Length of the Rule, as in this little Table annexed.

I.	0	0	0	0	5	0	8$\frac{1}{2}$	6	Long
F.	12	6	4	3	2	2	1	1	Long
I.	1	2	3	4	5	6	7	8	Broad

Here you ſee, if the Breadth be one Inch, the Length muſt be 12 Feet; if 2 Inches, the Length is 6 Feet; if 5 Inches broad, the Length is about 5 Feet 5 Inches, &c.

The reſt of the Lengths are expreſſed in the Line, thus: If the Breadth be 9 Inches, you will find it againſt 16 Inches, counted from the other End of the Rule: If the Breadth be 11 Inches, then a little above 13 Inches will be the Length of a Foot, &c.

§ II. *Of* SQUAR'D TIMBER.

BY ſquar'd Timber is here meant all ſuch as have Baſes, and the Sides ſtrait and parallel. The Rules for meaſuring all ſuch Solids are ſhewed in Sect. II. of Chap. 2. to which I refer you.

Example 1. If a Piece of Timber be 1 Foot 3 Inches (or 15 Inches) ſquare, and 18 Feet long, how many ſolid Feet are contained therein?

```
        15
        15
      ----                    F.  I.
        75                    1   3
       15                     1   3
      ----                  ----------
       225                    1   3
        18                        3   9
      ----                  ----------
      1800                    1   6   9
      225                             6
      ----                  ----------
144) 4050 (28.125             9   4   6
      ----                            3
      1170                  ----------
       180                   28   1   6
        360
         720
```

Anſwer. 28 Feet and half a Quarter.

Here, inſtead of multiplying by 18, (where I wrought by Feet and Inches) I muſt multiply by 6, and then by 3 becauſe 3 times 6 is 18.

Example 2. If a Piece of ſquar'd Timber be 2 Feet 9 Inches deep, and 1 Foot 7 Inches broad, and 16 Feet 9 Inches long, how many Feet of Timber are in that Piece?

Multiply the Depth, Breadth and Length together, and the Product will be the Content.

```
        33
        19
      ----
       297
       33
      ----
       627
     16.75
      ----
      3135
     4389
    3762
    627
    ----
144)10502.25 (72.93
     -----
    422
    1342
     465
    ----
     33
```

```
  F.  I.
   2   9
   1   7
  -------
   2   9
   1   7   3
  -----------
   4   4   3
  16   9
  -----------
  69   8   0
   3   3   2   3
  ---------------
  72  11   2   3
```

Anſwer. 72 Feet 11 Inches, or 72 Feet 93 Parts.

By Scale *and* Compaſſes.

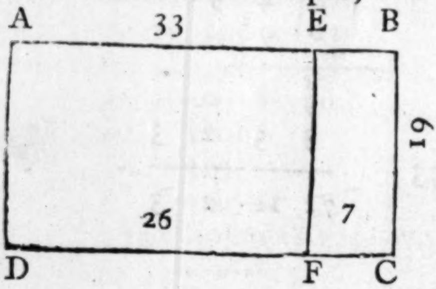
For the firſt *Example,* extend the Compaſſes from 12 to 15 Inches, (the Side of the Square) that Extent will reach from 18 Feet (the Length, being twice turned over) to 28 Feet, and ſomething more.

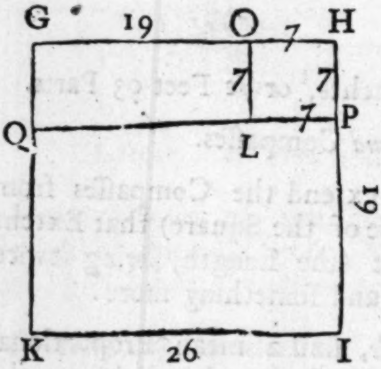
For the ſecond *Example,* find a mean Proportional between 19 Inches and 33 Inches, by dividing the Space between them into two equal Parts, and the Compaſs Point will reſt upon 25, which is a mean Proportional between 19 and 33.

Then extend the Compaſſes from 12 to 25, (the Proportional found) that Extent will reach (being twice turned over) from 16.75 Feet, the Length, to 72.93 Feet, the Content.

A common Error is committed, for want of Art, in meaſuring theſe laſt Sorts of Solids, by adding the

Depth and Breadth together, and taking Half for the Side of a mean Square. This Error, tho' it be but ſmall when the Depth and Breadth are pretty near equal, yet if the Difference be great the Error is very conſiderable; for the Piece of Timber, thus meaſured, will be more than the Truth, by a Piece, whoſe Length is equal to the Length of the Piece of Timber to be meaſured, and the Square equal to half the Difference of the Breadth and Depth, as I ſhall here demonſtrate.





I ſay, the Square GHIK is greater than the Parallelogram ABCD, by the little Square OHPL; for the Parallelogram QPIK is equal to the Parallelogram AEFD, and the Parallelogram GOLQ is equal to the Parallelogram EBCF. Therefore the Squa. is greater than the Parallelogram by the little Square OHPL. Which was to be proved.

Otherwiſe, you may prove it by Numbers, thus; the Sum of 33 and 19 is 52; the Half thereof is 26; the Square of 20 is 676; and the Product of the Depth and Breadth is 627; the Difference of theſe two is 49, equal to the Square of half the Difference; for the Difference between 33 and 19 is 14, the Hal thereof is 7, whoſe Square is 49. Which was to b proved.

Now, if this 49 be multiplied by the Length of th Piece, and that Product divided by 144, to bring it t Feet, and thoſe Feet added to the true Content, th Sum will be equal to the Content found by the fal Way mentioned.

See the Work of both.

33 Depth.	16.75 the Length.
19 Breadth	49 the Sq. of half Diff.
52 Sum	15075
	6700
26 Half.	
26	144)820.75(5.69
156	1007
52	1435
676	139
16.75	
3380	
4732	
4056	
676	
144)11323.00(78.63	
1243	
910	
460	
28	

Feet.
To 72.93 the true Content.
Add 5.69 the Part ſuperfluous

Sum 78.62 equal to the Content by the falſe Way.

By Feet *and* Inches.

F.	I.					F.	I.	
0	7					2	2	
0	7					2	2	
0	4	1				4	4	
16	9						4	4
5	5	4				4	8	4
	3	0	9			16	9	
5	8	4	9	Part ſuperfluous.		75	1	4
72	11	2	3	true Content add.		3	6	3
78	7	7	0	=Con. the falſ. way--Fal. Con.		78	7	7

To find how much in Length makes a Foot of any Squar'd Timber.

Always divide 1728 (the ſolid Inches in a Foot) by the Area of the Baſe, the Quotient is the Length of a Foot.

This Rule is general for all Timber, which is of equal Thickneſs from End to End, whether it be Square, Triangulár, Multangular, or Round.

Example 1. If a Piece of Timber be 18 Inches ſquare, how much in Length will make a Foot ſolid?

```
  18
  18
 ———
 144
 18
 ———
 324)1728(5⅓
     ————
     1620
      108
```

Anſw. 5 Inches and one Third.

By

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 18, that Extent will reach from 18 to 324, the Square or Area of the Baſe; then extend from 324 to 1728, that Extent will reach down from 1 to 5 Inches and $\frac{1}{3}$ of an Inch.

Or thus: Extend the Compaſſes from 18 to 41.569, that Extent (turned twice over from 1) will at laſt fall upon 5 $\frac{1}{3}$ as before.

NOTE, That 41.569 is the ſquare Root of 1728.

Example 2. If a Piece of Timber be 22 Inches deep, and 15 Inches broad, how much in Length will make a Foot.

22
15
———
110
22
———
330) 1728 (5.23
———
780
1200
———
210

Anſwer. 5 Inches and 23 Parts.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 15, that Extent will reach from 22 to 330; then extend from 330 to 1728, that Extent will reach from 1 to 5.23 Inches, the Length of a Foot.

There is a Line for this Purpoſe upon moſt ordinary Rules, with a little Table at the End, of all ſuch Numbers as exceed the Length of the Rule ſuch as this annexed.

0	0	0	0	9	0	11	3	9	Inches.
144	36	16	9	5	4	2	2	1	Feet.
1	2	3	4	5	6	7	8	9	Side of Square

Here you ſee, if the Side of the Square be 1, the Length muſt be 144 Feet; if two Inches be the Side of the Square, it muſt be 36 Feet in Length, to make a ſolid Foot, *&c.*

If the Side of the Square be not in the little Table, you will find it upon the Line, thus; if the Side of the Square be 16 Inches, you will find it againſt 6 Inches and 7 Tenths, counted from the other End of the Rule.

Then, if you take the Length of a Foot from the Line of Inches with your Compaſſes, and run the Compaſſes along the Piece from End to End, you will find how many Feet are contained in that Piece, or you may cut off any Number of ſolid Feet that ſhall be deſir'd; but if the Sides of the Piece be unequal, find a mean proportional Number, as is before taught, by dividing the Diſtance upon the Line of Numbers into two equal Parts, thus; if the Breadth be 25 Inches, and the Depth 9 Inches, divide the Space upon the Line of Numbers into two equal Parts, and you will find the middle Point at 15, ſo is 15 Inches the geometrical mean Proportional ſought; then, if you look for 15 upon the Line above mentioned, that 7 Inches, and a little above half, will be the Length of a Foot.

§ III. *Of unequal* Squar'd Timber.

BY unequal ſquar'd Timber, I mean all ſuch as have unequal Baſes, that is, ſuch as is thicker at one End than at the other; and ſuch are moſt Timbe

Trees, when they are hewn, and brought to their Squares.

The uſual Way to meaſure ſuch Timber, is to take a Square about the Middle of the Piece, which they take to be a mean Square: This Way, when the Piece is pretty near as thick at one End as the other, is ſomething near the Truth; but when there is a great Diſproportion between the Ends of the Piece, the Error is conſiderable. All ſuch Solids being the Fruſtums of Pyramids, the true Way of meaſuring them muſt be by Sect. VII. Chap. 2. I ſhall give an *Example* or two, which I will work both by the true and falſe Ways, whereby you will ſee the Difference.

Example 1. If a Piece of Timber be 25 Inches ſquare at the greater End, and 9 Inches ſquare at the leſſer End, and 20 Feet long, how many Feet of Timber are in that Tree?

```
        25
         9
        ——
Sum     34
        ——
Half.   17 The Side of the Square in the Middle.
        17
        ——        By Rule II. Sect. VII. Chap. 2.
       119
        17           25          25
        ——            9           9
       289          ———          ——
        20          225          16 Diff. of the Sides.
        ——                       16
144) 5780(40.13                  ——
        ——                       96
      .200                       16
       560                       ——
      ————                   3) 256 The Square.
       128                       85.3333
                                225
                               ————
                               310.3333
```

Anſwer. 40.13 Feet by the falſe Way.

310

```
      310.3333
            20
      --------
144)6206.6660(43.101
      --------
      446
       146
        266
       ---
       122
```

Anſwer. 43.101 Feet by the true Way; ſo that there is near three Feet Difference.

By Scale *and* Compaſſes.

Extend from 1 to 9, that Extent will reach from 25 (the ſame Way) to 225; the Rectangle of the Sides of the two Baſes; then the Difference between the ſaid Sides is 16: Extend from 3 to 16, that Extent will reach from 16 to 85.333, a third Part of the Square; which added to 225, the Sum is 310.333, a mean Area: Then extend from 144 to 310.333, that Extent will reach from 20 (the Length) to 43.1 Foot (the Content) the true Way.

Extend the Compaſſes from 12 to 17 (the Side of the middle Square) that Extent will reach from 20 (the Length being twice turned over) to 40. 1 Feet, the Content by the falſe Way.

Example 2. If a Piece of Timber be 32 Inches broad and 20 Inches deep at the greater End, and 10 Inches broad and 6 deep at the leſſer End, and 18 Feet long, how many Feet of Timber are in that Piece?

Rule I. Sect. VII. Chap. 2

```
   32                    6
   20                   10
  ----                 ----
  640                   60
   60
 -----
 .....
 38400                 38400
```

38400 ……… (195.959 Mean Proportional.

1 640. The greater Baſe.

60. The leſſer Baſe.

29)284

385)2300 895.959 The Sum

3909)37500 6=$\frac{1}{3}$ Height.

39185)231900

391909)3597500 5375.754

144)5375.754(37.33

1055

477

455

23

Add.	32 10			20 6	Add.
Sum.	42			26	Sum.
Half.	21			13	Half.

13

63

21

273 Area in the Middle.

18 Length.

2184

273

144)4914(34.12

594

180

360

72

		Feet.
Anſwer.	Content the true Way	37.33
	Content the falſe Way	34.12

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 20, that Extent will reach from 32 to 640, the Area of the greater Baſe.

Then extend from 1 to 10, that Extent will reach from 6 to 60, the Area of the leſſer Baſe. Then extend from 1 to 60, that Extent will reach from 640 to 38400, the Product of the two Areas: Find the Square Root thereof, by dividing the Space between 1 and 38400 into two equal Parts, ſo you will find the middle Point at 195.959, the Root ſought; which is a mean Proportional between the greater and the leſſer Area. Then add the mean Proportional and two Areas together, and the Sum is 895.959; which multiply by 6 (a third Part of the Length) by extending from 1 to 6, that Extent will reach from 895.959 to 5375.75, Then extend from 144 to 5375.75, and that Extent will reach from 1 to 37.33 Feet, the true Content.

For the falſe Way, half the Sum of the Breadths is 21, which is the Breadth in the Middle; and half the Sum of the Depths is 13: Extend from 1 to 13, that Extent will reach from 21 to 273, the Area of the middle Baſe. Then extend from 144 to 273, that Extent will reach from 18 (the Length) to 34.12. the Content the falſe Way.

---

§ IV. *Of* Round Timber, *whoſe Baſes are equal.*

THE uſual Way to meaſure round Timber Trees, is to girt them about the Middle with a String, and take the fourth Part of that Girt for the Side of a Square, by which they meaſure the Piece of Timber as if it was ſquare.

But that this is an Error, I ſhall make appear as follows: If the Circumference of the Circle be 1, the Area will be .07958; then the fourth Part of 1 is

.25, which ſquared makes .0625, this they take for a mean Area inſtead of .07958: Therefore the true Content always bears ſuch Proportion to the Content found by the aforeſaid cuſtomary falſe Way, as .07958 to .0625: which is nearly as 23 to 18, ſo that in meaſuring by that cuſtomary falſe Way, there is above the one fifth Part loſt of what the true Content ought to be.

This Error, tho' it has been ſo often confuted, yet it is going ſo cuſtomary in all Places, that there is little Hopes of my prevailing with Men that are ſo wedded to it, to embrace the Truth; I ſhall therefore, in the following *Example*, ſhew how to work both the true Way, and alſo the falſe or cuſtomary Way.

Example 1. If a Piece of Timber be 96 Inches in Circumference, or Girt, and 18 Feet long, how many Feet of Timber are contained therein?

A fourth Part of 96 is 24
24
96
48
576 Area Baſe.
18
4608
576
144) 10368 (72
1008
288
288
. . .

Or thus.

F.	I.
2	0
2	0
4	0
18	
72	0

Content the falſe Way 72 Feet.

Then the true Way.

96
96

576
864

9216
.07958

73728
46080
82944
64512

733.40928 The Area by Prob. 5. Sect. IX,
18 (Ch. 2,

586727424
73340928

144)13201.36704(91.67

241 The true Content 91.67 Feet,
973
1096

88

By Scale *and* Compaſſes.

Extend from 12 to 24 (the fourth Part of the Gir that Extent turned twice over from 18 Feet (th Length) will at laſt fall upon 72 Feet, the Content th cuſtomary Way.

Extend from 42.54 to 96, (the Girt) that Exten will reach from 18 Feet turned twice over, to 91.6 Feet, the true Content.

Examp

Example 2. If a Piece of Timber be 86 Inches Girt, and 20 Feet long, how many Feet are contained therein?

The fourth Part of 86 is 21.5
21.5
1075
215
430
462.25
20
144)9245.00(64.2
605
290
20

F.	I.	P.	
1	9	6	
1	9	6	
1	9	6	
1	4	1	6
	0	10	9
3	2	6	3
		20	
64	2	5	0

The Content the falſe Way 64.2 Feet.

By the true Way.

86
86
516
688
7396

7396

7396
.07958

59168
36980
66564
51772

588.57368
20

144)11771.47360(81.74

251
1074
667

91

The true Content 81.74 Feet.

By Scale *and* Compaſſes.

Extend from 12 to 21.5, that Extent turned twice over, from 20, will reach at laſt to 64.2 Feet, the Content the falſe Way.

Extend from 42.54 to 86, that Extent turned twice over from 20, will at laſt fall upon 81.74 Feet, the true Content.

Theſe Cylindrical Proportions may be very eaſily wrought upon the Line of Numbers.

Problem 1. Having the Diameter of a Cylinder in Inches, to find the Length of a Foot.

Suppoſe the Diameter is 22.6 Inches.

As 22.6 : to 46.9 : : ſo is 1 to a fourth Number, and that to the Length of a Foot in Inches, 4.3

Extend the Compaſſes from 22.6 to 46.9, that Extent will reach from 1 to a fourth Number, then turn them over again, and that will reach to 4.3 Inches.

Problem 2. Having the Diameter in Foot-meaſure, to find the Length of a Foot in Foot-meaſure.

Suppoſe the Diameter 1.88 Feet.

Then, as 1.88 : to 1.128 : : ſo is 1 : to a fourth Number ; and ſo is that to the Length of a Foot in Foot meaſure 358.

Extend the Compaſſes from 1.88 to 1.128, that Extent turned twice from 1, will reach to 358 Parts of a Foot.

Problem 3. Having the Circumference in Inches, to find the Length of a Foot in Inches.

Suppoſe the Circumference 71 Inches.

Then, as 71 : to 147.36 : : ſo is 1 to a fourth Number, and ſo is that to the Length of a Foot in Inches 4.3

Extend the Compaſſes from 71 to 117.36, that Extent turned twice from 1, will reach to 4.3 Inches, the Length of a Foot.

Problem. 4 Having the Circumference in Foot-meaſure, to find the Length of a Foot in Foot-meaſure.

Suppoſe the Circumference 5.92 Feet.

Then, as 5.92 : to 3.545 : : ſo is 1 : to a fourth Number ; and ſo is that to the Length of a Foot in Foot-meaſure .358.

Extend the Compaſſes from 5.92 to 3.545, that Extent turned twice over from 1, will fall upon .358 Parts of a Foot.

Problem 5. Having the Diameter in Inches, and Length in Inches, to find the Content in Inches.

Suppoſe the Diameter is 22.6 Inches, and the Length is 156 Inches, or 13 Feet.

Then, as 1.128 : to 22.6 : : ſo is 156 : to a fourth Number; and ſo is that to the Content in Inches 62674.

Extend the Compaſſes from 1.128 to 22.6, that Extent turned twice from 156, will fall upon 62674 Inches, the Content.

NOTE, That 1.128 is the Diameter when the Side of the Square equal is = 1.

Problem 6. Having the Diameter in Foot-meaſure, and Length in Feet, to find the Content in Feet.

Suppoſe the Diameter 1.88 Feet; and the Length 13 Feet.

Then, as 1.128 : to 1.88 : : ſo is 13 to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend from 1.128 to 1.88, that Extent turned twice from 13, will fall upon 36.27.

Problem 7. Having the Diameter in Inches, and Length in Inches, to find the Content in Feet.

Suppoſe the Diameter 22.6 Inches, and the Length 156 Inches.

Then, as 46.9 : is to 22.6 : : ſo is 156 : to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend from 46.9 to 22.6 that Extent turned twice from 156, will fall upon 36.27 Feet, the Content.

NOTE, That 46.9 is the Diameter of a Circle whoſe Area is 1728.

Problem 8. Having the Diameter in Inches, and Length in Feet, to find the Content in Feet.

Suppoſe the Diameter 22.6 Inches, and the Length 13 Feet.

Then, as 13.54 : to 22.6 : : ſo is 13 to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend from 13.54 to 22.6, that Extent turned twice from 13, will fall upon 36.27

NOTE, That 13.54 is the Diameter of a Circle when the Area is 144.

Problem 9. Having the Circumference in Inches, and Length in Inches, to find the Content in Inches.

Suppoſe the Circumference 71, and the Length 156 Inches.

Then, as 3.545 : is to 71 : : ſo is 156 to a fourth Number; and ſo is that to 62674, the Content in Inches.

Extend the Compaſſes from 3.545 to 71, that Extent turned twice from 156, will fall upon 62674, the Content.

NOTE, That 3.545 is the Circumference when the Side of the Square is equal 1.

Problem 10. Having the Circumference in Feet, and Length in Feet, to find the Content in Feet.

Suppoſe the Circumference 5.92 Feet, and Length 13 Feet.

Then, as 3.545 : to 5.92 : : ſo is 13 : to a fourth Number; and ſo is that to 36.27.

Extend from 3.545 to 5.92, that Extent turned twice from 13, will fall upon 36.27 Feet, the Content.

Problem 11. Having the Circumference in Inches, and Length in Inches, to find the Content in Feet.

Suppoſe the Circumference 71 Inches, and Length 156 Inches.

Then, as 147.36 : to 71 : : ſo is 156 to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend the Compaſſes from 147.36 to 71, that Extent turned twice from 156, will fall upon 36.27 Feet, the Content.

NOTE, That 147.36 is the Circumference of a Circle whoſe Area is 1728.

Problem 12. Having the Circumference in Inches, and Length in Feet, to find the Content in Feet.

Suppoſe the Circumference 71 Inches, and Length 13 Feet.

Then, as 42.54 : is to 71 : : ſo is 13 : to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend the Compaſſes from 42.54 to 71, that Extent turned twice from 13, will reach to 36.27 Feet, the Content.

NOTE, That 42.54 is the Circumference of a Circle whoſe Area is 144.

§ V. *Of* Round Timber, *whoſe Baſes are unequal.*

THE uſual Way to meaſure Round Timber (as I ſaid before) is to take a fourth Part of the Girt in the Middle of the Piece, for the Side of a mean Square. But this Way I have proved to be erroneous in Timber that is all the Way of an equal Thickneſs; and it muſt be much more ſo in Timber that is tapering, and the more tapering it is the greater is the Error: For to the Error in the laſt Section, there is added the Error in the third Section; therefore, to meaſure all ſuch Timber according to Art and Truth, ſuch a Piece ought to be conſidered as a Fruſtum of a Cone, and ſhould be meaſured by the Rules given in Sect. VIII. Chap. 2. by which Rules the following *Examples* are wrought.

Example 1. If a Piece of Timber be 9 Inches Diameter at the leſſer End, and 36 Inches at the other End, and 24 Feet long, how many Feet of Timber are therein?

36   36 } Subtract.
9   9 }
Rect. 324   27 Difference.
27
189
54
3) 729 The Square.
243 One Third.
324 Rectangle add.
567
.7854
567
54978
47124
39270
A mean Area 445.3218
24
17812872
8906436
144)10687.7232(74.22
607
317
292
4 *Anſwer.* 74.22 Feet.

Or thus, by Feet and Inches.

F.	I.		F.	I.		
3	0		2	3		Difference.
0	9		2	3		
2	3	Reſt.	4	6		
			0	6	9	
			5	0	9	the Square.
			1	8	3	One Third.
			2	3	0	Rectangle added.
			3	11	3	A mean Square.

F. I. P.

Then, as 14 : to 11 : : ſo is 3 11 3 to the Area.

11

7)43 3 9

2) 6 2 3

3 1 1 6

6

18 6 9 0

4

74 3 0 0

Here, inſtead of dividing by 14, I divide by 7 and by 2, becauſe twice 7 is 14.

And inſtead of multiplying by 24 Feet, the Length, I multiply by 6 and by 4, becauſe 6 times 4 is 24.

By Scale and Compaſſes this is too troubleſome.

Example 2. If a Piece of Timber be 136 Inches Circumference at one End, and 32 Inches Circumference at the other End, and 21 Feet long, how many Feet of Timber are contained in that Piece?

136

136 136
32 32

272 104 Difference.
408 104

4352 416
1040

3)10816 the Square.

3605.333 one Third.
4352 Rectangle add.

7957.333 a mean Circumf. ſquared.
.07958

63658664
39786665
71615997
55701331

633.24456014 the mean Area.
21

633.24456014
126648912028

13298.13576294

144)13298.13(92.34

338
501
693

117

Anſwer. 92.34 Feet.

By Feet *and* Inches *thus.*

F.	I.			F.	I.			
11	4			8	8			Difference.
2	8			8	8			
22	8			69	4			
7	6	8		5	9	4		
30	2	8		3)75	1	4		the Square.
				25	0	5	4	
				30	2	8	0	
				55	3	1	4	Squ: of the Circumfer.

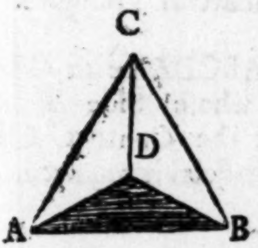
	F	I	P	S	
As 88 : to 7 ::	55	3	1	4	: to the mean Area.
				7	
11)	386	9	9	4	
8)	35	1	11	9	
	4	4	8	11	The mean Area
				7	
	30	9	2	5	
				3	
Facit	92	3	7	3	

§ VI.

§ VI. *Of the five* Regular Bodies.

THESE Bodies may all be meaſured by the fourth Section of Chap. II. except it be the Cube, or Hexaedron, which is already meaſured in Sect. I. of that Chapter.

1. *Of the* TETRAEDRON.



A Tetraedron is a Solid contained under four equal and equilateral Triangles.

Let ABCD be a Tetraedron, whoſe Side is 12 Inches, and the perpendicular Height, 9,798 Inches.

By Sect. V. Chap. I. the Area of the Triangle will be found 62.352 ; a third Part of it is 20.784, which multiplied by the Perpendicular Height, the Product is 203.641632 ſolid Inches, the Content.

10.392 The Perpendicular of the Triangle.
6 Half the Side.

62.352 Area of the Triangle.

---

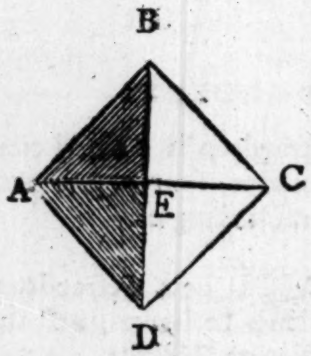
20 784 One third Part.
9.798 The Perpendicular Height.

166272
187056
145488
187056

203.641632 The Solidity.

The ſuperficial Content is four times the Area of the Triangle, *viz.* 249.408 Inches becauſe there are 4 Triangles.

2. *Of the* OCTAEDRON.



The Octaedron is a Body contained under eight equal and equilateral Triangles.

Let ABCDE be an Octaedron whoſe Side is 12 Inches; the Content ſolid and ſuperficial is required.

An Octaedron is compoſed of two quadrangular Pyramids joined together by their Baſes; therefore, if the Area of the Baſe be multiplied into a third Part of the Length of both Pyramids, the Product will be the ſolid Content.

5.6568 A third Part of the Length.
144 Area of the ſquare Baſe.

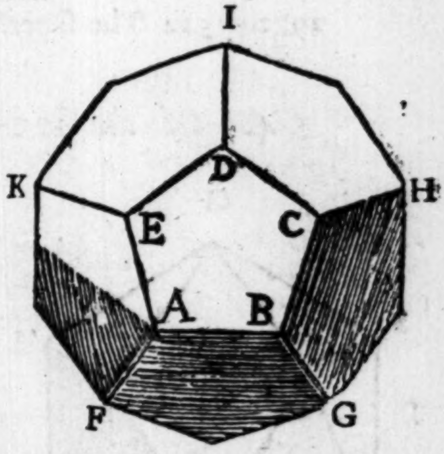
226272
226272
56568

814.5792 The Solidity.

The ſuperficial Content will be juſt double to that of the Tetraedron, *viz.* 498.816, becauſe the Side of this is ſuppoſed to be equal to the Side of that, and becauſe the Octaedron is contained under eight Triangles, and the Tetraedron but under four.

3. *Of the* DODECAEDRON.

The Dodecaedron is a ſolid Body, contained under twelve pentangular Planes.



Let ABCDEFGHIK be a dodecaedron each Side thereof being 12 Inches; the Content ſolid and ſuperficial is required.

The Solidity of the Dodecaedron is compoſed of 12 pentangled Pyramids, whoſe Vertices all meet in the Center. Therefore if we find the Solidity of one of thoſe Pyramids, and multiply that by 12, that Product will be the Solidity of the Dodecaedron.

The Altitude of one of the pentangled Pyramids will be found to be 13.36219.

The Perpendicular of the Pentagon will be

8.258292
30 Half the Sum of the Sides.

247.748760 Area of the Pentagon.
60454.4 A third Part of 13.36219 inverted.

99099504
9909950
1238744
99099
1486

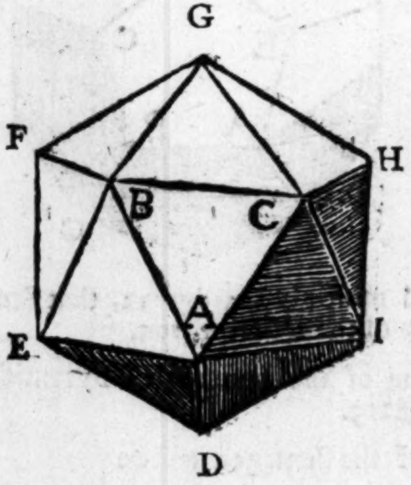
1103.48783 Content of one Pyramid.
12

13241.85396 The Solidity of the Dodecaedron.

If the Area of the Pentagon be multiplied by 12, the Product will be the ſuperficial Content.

```
247.74876
       12
─────────────
2972.98512 The ſuperficial Content.
```

4. *Of the* ICOSAEDRON.



The Icoſaedron is a ſolid Body, contained under 20 equal and equilateral Triangles.

Let ABCDEFGHI be an Icoſaedron, each Side thereof being 12 Inches; the Content ſolid and ſuperficial is required.

The Icoſaedron is compoſed of twenty triangular Pyramids, with their Vertices all joining in the Centre.

Therefore, if the ſolid Content of one Pyramid be multiplied by 20, the Product is the whole ſolid Content of the Icoſaedron.

10.39224

10.39224 The Perpendicular of the Triangle.
6 Half the Side.

62.35344
20

1247.06880

3.0230456 The third Part of the Altitude of the (Pyramid.
44353.26

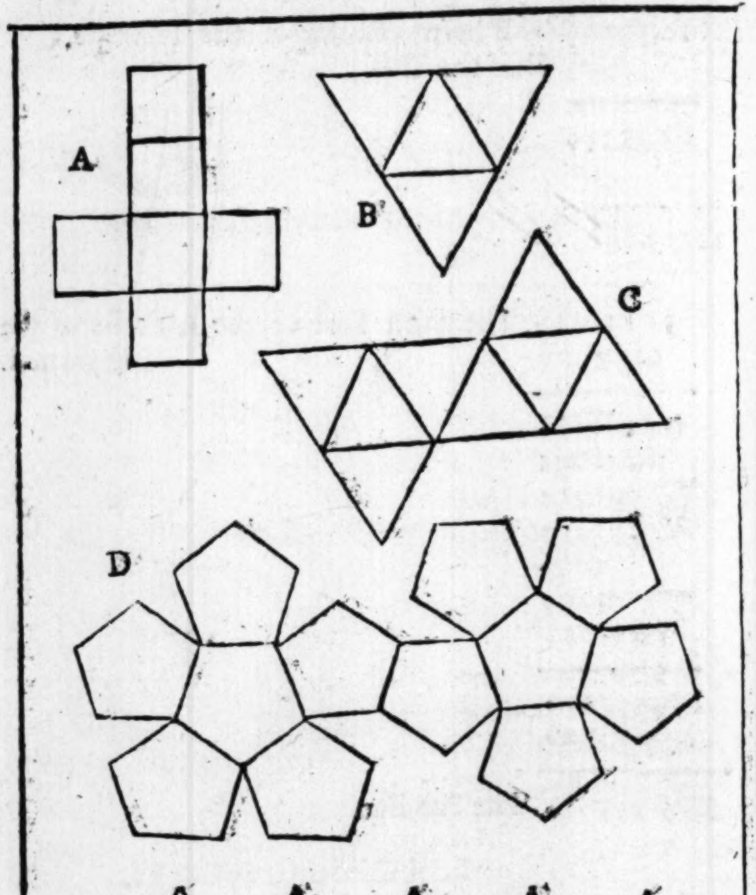
181382736
6046091
906914
151152
9069
1209
121

188.497292
20

3769.945840 The Solidity.

The ſuperficial Content 1247.0688.

A the



A the Cube: B Tetraedron. C Octaedron.
D Dodecaedron. E Icoſaedron.

By theſe Figures you may cut theſe Bodies in fine Paſtboard, cutting all the Lines half through, and ſo turn them up and glue them.

A TABLE *ſhewing the Solidity and ſuperficial Content of any of the regular Bodies, the Side being* 1, *or Unity.*

	The Names of the Bodies.	Solidity.	Superficies.
B	Tetraedron	0.1178511	1.732051
C	Octaedron	0.4714045	3.464102
A	Hexaedron	1.0000000	6.000000
E	Icoſaedron	2.181695	8.660254
D	Dodecaedron	7.663119	20.645729

By this Table the Content, either ſuperficial or ſolid, of any of theſe Bodies may very readily be found; for all like ſuperficial Figures are in proportion one to another, as are the Squares of their like Sides: Therefore it will be, as the Square of 1 (which is 1): is to the ſuperficial Content in the Table, : : ſo is the Square of the Side of the like Body: to the ſuperficial Content of the ſame Body. Therefore, if the Number in the Table be multiplied by the Square of the Side given, the Product is the ſuperficial Content required.

Again. All like Solids are in ſuch Proportion to each other as are the Cubes of their like Sides; therefore it will be, as 1: (which is the Cube of 1) is to the ſolid Content in the Table : : ſo is the Cube of the Side given : to the ſolid Content required. Therefore, if the Number in the Table be multiplied by the Cube of the given Side, the Product will be the ſolid Content of the ſame Body.

Example 1. If the Side of a Dodecaedron be 12 Inches (as before) what is the Content ſolid and ſuperficial?

7.663119 The tabular Number.
1728 The Cube of the Side.

61304952
15326238
53641833
7663119

13241.869632 The ſolid Content nearly the ſame as before.

20.645729 The tabular Number.
144 The Square of the Side.

82582916
82582916
20645729

2972.984976 The ſuperficial Content.

By Scale *and* Compaſſes.

Extend from 1 to 12, (the Side,) that Extent being turned three times over from 7.663119, will at laſt fall upon 13241.86, *&c.* the ſolid Content.

And if you apply the ſame Extent twice from 20.645729, it will at laſt fall upon 2972.98, *&c.* the ſuperficial Content.

Example 2. If the Side of an Octaedron be 20 Inches, what is the Content ſolid and ſuperficial?

.4714045 The tabular Number.
8000 The Cube of the Side.

3771.2360000 the ſolid Content.

3.464102 The tabular Number.
400 The Square of the Side,

1385.640800 The ſuperficial Content.

By

By Scale *and* Compaſſes.

Extend from 1 to 20, that Extent turned three times over from .4714045, will at laſt fall upon 3771,236, the ſolid Content. The ſame Extent turned twice over from 3.464, &c. will at laſt fall upon 1385.64, the ſuperficial Content.

---

§ VII. *How to meaſure any irregular Solid.*

IF you have any Piece of Wood or Stone that is craggy and uneven, and you deſire to find the Solidity, put the Solid into any regular Veſſel, as a Tub, a Ciſtern, or the like, and pour in as much Water as will juſt cover it; then take out the Solid, and meaſure how much the Fall of the Water is, and ſo find the Solidity of that Part of the Veſſel.

Example. Suppoſe a Piece of Wood or Stone to be meaſured, and ſuppoſe a Tub 32 Inches Diameter, into which let the Stone or Wood be put and covered with Water; then when the Solid is taken out, ſuppoſe the fall of the Water 14 Inches; ſquare 32, and multiply the Square by .7854, the Product will be 804.2496, the Area of the Baſe; which multiplied by 14, the Depth or fall of Water, and the Product is 11259.49, &c. which divided by 1728, the Quotient is 6.51 Feet; and ſo much is the ſolid Content required.

H

CHAP.

CHAP. V.

Practical Queſtions in MEASURING.

Queſtion 1. IF a pavement be 47 Feet 9 Inches long, and 18 Feet 6 Inches broad, I demand how many Yards are contained therein?

F.	I.		
47	9		47.75
18	6		18.5
376	0		23875
47			38200
23	10	6	4775
9	0	0	
4	6	0	9)883.375
9)883	4	6	98.1

Anſwer. 98 Yards 1 Foot.

Queſt. 2. There is a Room whoſe Length is 21.5 Feet, and the Breadth 17.5 Feet, which is to be paved with Stones each 18 Inches ſquare; I demand how many ſuch Stones will pave it?

21.5 1.5
17.5 1.5

1075 75
1505 15
215

2.25 Area of one Stone.

2.25)376.25(167

1512
1625

50 *Anſw.* 167

Queſt. 3. There is a Room 109 Feet 9 Inches about and 9 Feet 3 Inches high, which is all (except tw Windows, each 6 Feet 6 Inches high, and 5 Fee 9 Inches broad) to be hung with Tapeſtry that i Ell broad; I deſire to know how many Yards wil hang the ſaid Room.

From the Content of the Room ſubtract the Content of the Windows, and divide the Remainder by the ſquare Feet in a Yard of Tapeſtry.

3.73 109.75 Length. 5.75
3 9.25 Breadth. 6.5

11.25 54875 2875
21950 3450
98775

373 5
1015.1875 Content of the Room. 2
74.75 Cont. of the Windows ſubt.

74.750

11.25)940.4375(83.59
4043
6687
10625

500 *Anſwer.* 83. 59 Yards.

N *Queſt.*

Queſt. 4. If the Axis of a Globe be 27.5 Inches, I demand the Content ſolid and ſuperficial.

3.1416
27.5

157080
219912
62832

86.39400 The Circumference.
27.5 The Diameter.

431970
604758
172788

6)2375.8350 The ſuperficial Content

395.9725 A ſixth Part.
27.5

19798625
27718075
7919450

10889.24375 The Solidity in Inches.

Anſwer. } 6.3 Feet ſolid.
16.49 Feet ſuperficial.

Queſt. 5. There is the Fruſtum of a Globe, the Diameter of whoſe Baſe is 24 Inches, and the Altitude thereof is 10 Inches; what is the Content ſolid and ſuperficial?

Find the Superficies, as is directed in *Page* 177 and find the Solidity by the firſt Theorem in *Page* 188

24	.7854	.7854	20
24	576	400	20
96	47124	314.16000	400
48	54978		
576	39270		
	452.3904 } add		7
	314.16 }		

766.5504 The Curve-Superficies.
452.3904 The Baſe add.

1218.9408 The whole ſuperficial Content in Inches.
12×12=144
3

432
100 The Square of the Altitude add.

532 The Sum.
10 Multiply by the Altitude.

5320
.5236 Multiply.

31920
15960
10640
26600

2785.5520 The Solidity in Inches.

Queſt. 6. If a Tree girt 18 Feet 6 Inches, and be 24 Feet long, how many Tuns of Timber are contained in that Tree?

F. I.
4)18 6 The Girt.

4 7 6 A 4th Part.
4 7 6

18 6 0
2 8 4 6
2 3 9

21 4 8 3
6

128 4 1 6
4

4|0 51|3 4 6 0

12 33

Here I multiply by 6 and by 4, becauſe 6 times 4 is 24.

Anſwer. 12 Tuns, 33 Feet, 4 Inches, 6 Parts.

Note, That 40 Feet of Timber is a Tun, and 50 Feet a Load.

Note alſo, That 4 Feet broad, 4 Feet deep, and 8 Feet long, is a Cord of Firewood, that is, 128 cubical Feet.

Queſt. 7. There is a Cellar to be dug by the Floor, whoſe Length is 33 Feet 7 Inches, and the Breadth is 18 Feet 9 Inches, and its Depth is to be 5 Feet 9 Inches; I demand how many Floors of Earth are in that Cellar?

F.	I.		
33	7		The Length.
18	9		The Breadth.
264			
33			
16	9	6	
8	4	9	
9	0	0	
1	6	0	
629	8	3	
5	9		The Depth.
3148	5	3	
314	10	1	
157	5	0	
324)3620	8	4(11	
380			
56			

Anſwer. 11 Floors, 56 Feet.

Note, That 18 Feet ſquare and a Foot deep is a Floor of Earth, that is 324 ſolid Feet.

Queſt. 8. There is a Roof covered with Tiles, whoſe Depth on both Sides (with the uſual Allowance at the Eaves) is 35 Feet 6 Inches, and the Length 48 Feet 9 Inches; how many Squares of Tiling are contained therein?

```
   F.   I.
   48   9
   35   6
 ----------
  240
 144
   24   4 · 6
   17   6   0
    8   9   0
 --------------
 17|30  7   6      Anſw. 17 Squares, 30 Feet.
```

Queſt. 9. There is a Cone, whoſe Diameter at the Baſe is 42 Inches, and the perpendicular Height 94 Inches, and it is required to cut off two ſolid Feet from the Top End thereof; I demand what Length upon the Perpendicular muſt be cut off?

```
           42      1722         94
           42         2         94
         ----    ------       -----
           84      3456        376
          168                  846
         ----                 -----
         1764                 8836 Square.
         7854                   94
        -----                ------
         7056                35344
        8820                79524
     14112                  -------
    12348                   830584 The Cube.
   -----------
    1385.4456
           94
   -----------
     55417824
    124690104
   -----------
 3)130231.8864
   -----------
    43410.628
```

All the ſolid Bodies are in triplicate Reaſon of their homologous Sides by *Eucl.* 12, 12; 12, 18; and 11, 33; therefore it will be,

Sol. of the Cone. Cube Alt. Sol. of 2 Feet.
As 43410.6288 : 830584 :: 3456 :
3456

4983504
4152920
3322336
2491752

43410.6288)2870498304(66124 The Cube of the Len.
....

265860576
5396803
1055740
187528

66124(40.43
64

2124000 Reſolvend.

12
48

492 Diviſor.

120
4800

48120 Diviſor.
64
1920
19200

1939264 Subtrahend.

184736000 Reſolvend.

1212
489648

4897692 Diviſor.

27
10908
1468944

147003507 Subtrahend.

37732493

Anſwer. The Length upon the Perpendicular muſt be 40.43 Inches. If it had been 3 Feet, the Length had been 46.29 Inches.

If two Feet were to be cut off from the Bottom, or greateſt End, then from 43410.6288 ſubtract 3456, and the Remainder is 39954.6288. Then ſay,

As 43410.6288 : 830584 : : 39954.6288
830584

1598185152
3196370304
1997731440
1198638864
3196370304

13410.6288)33185675407.2192(764459(91.4
. ....
279823524
19359751
1995500
259075
42029

2952

729

35459

27
243

2457

2457

271
243

24571

10888000

273
24843
248703

Anſwer. It muſt be cut at 91.4 Inches from the Top, or 2.6 Inches from the Bottom.

Queſt. 10. If a ſquare Piece of Timber be 12 Feet long, and if the Side of the Square of the greater Baſe be 21 Inches, and the Side of the Square of the leſſer Baſe be 3 Inches; how far muſt I meaſure from the greater End to cut off 5 ſolid Feet?

Firſt, find the Length of the whole Pyramid, thus; the Difference between 21 and 3 is 18; then,

	Diff.	Length.		great.	Length.
As	18 :	12	::	21 :	14

So I find the whole Length of the Pyramid 14 Feet, or 168 Inches.

The ſolid Content of the whole Pyramid is 24696 Inches, and the ſolid Content of 5 Feet is 8640; which ſubtracted from 24696, there remains 16056 Inches. Then the Cube of 168 (the length) is 4741632. Then,

As 24696 : 4741632 : : 16056 :

To 3082752, whoſe Cube Root is 145.54; ſubtract this Root from 168 (the Length) and there remain 22.46 Inches, which is the Length of 5 ſolid Feet at the greater End.

Queſt. 11. Three Men bought a Grinding Stone of 40 Inches Diameter, which coſt 20 Shillings; of which Sum the firſt Man paid 9 Shillings, the ſecond 6 Shillings, and the third 5 Shillings; I demand how much of the Stone each Man muſt grind down, proportionable to the Money he paid?

All Circles are in duplicate Reaſon of their Diameters, by *Eucl.* 12. 2.

Square the Semidiameter, which makes 400. Then.

And as 20 *s.* : 400 :: 9 *s.* : 180.

This 180 is the Square of the Semidiameter of the Circle belonging to the firſt Man.

As 20 *s.* : 400 :: 6 *s.* : 120.

This 120 is the Square of the Semidiameter of the Circle belonging to the ſecond.

And, as 20 *s.* : 400 :: 5 *s.* : 100.

This 100 is the Square of the Semidiameter of the Circle belonging to the third.

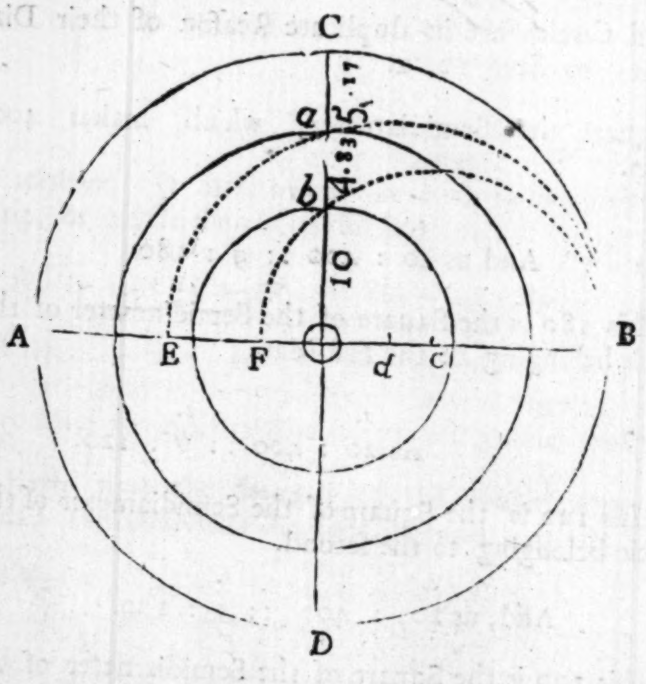
Then, from 400, (the Square of the Semidiameter of the Stone) ſubtract 180, and there remains 220, whoſe ſquare Root is 14.83 Inches; which ſubtracted from 20 Inches (the Semidiameter) there remains 5.17 Inches, which is the Breadth of the Ring, or Part of the Stone which muſt be ground down by the firſt.

Then, from 220 ſubtract 120, and there remains 100, whoſe ſquare Root is 10: ſubtract that from 14.83, and there remains 4.83 Inches, the Breadth

of

of the Ring, or Part to be ground down by the ſecond Man. The third ~~muſt~~ grind down the Remainder, which is 10 Inches, the ſquare Root of 100.

This Queſtion may very eaſily and ſpeedily be performed geometrically, as in the annexed Scheme.



Firſt, upon the Center ⊙ ſtrike the Circle ACBD, and croſs it at right Angles with the two Diameters AB and CD: Then divide the Semi-diameter A ⊙ (which ſuppoſe 20) in Proportion, to 9*s.* 6*s.* and 5*s.* (the ſeveral Sums paid by the three Men) by the Points E and F; ſo ſhall AE be 9, EF 6, and F⊙5: Then divide EB into two equal Parts in *d*, and upon *d*, as a Center, ſtrike the Semi-circle E*a*B, and divide FB into two equal Parts in *c*, and upon *c*, as a Center with the Radius *c*F, ſtrike the Semicircle F*b*B: So have you the Semi-diameter ⊙ C divided into three ſuch Parts as the Stone ought to be divided; and Circles ſtruck thro' theſe Points will ſhew how much each Man muſt grind for his Share.

Queſt. 12. A Gard'ner he had an upright Cone,
Out of which ſhould be cut him a Roling-ſtone.
The biggeſt that e'er it could make:
The Maſon he ſaid, that there was a Rule
For ſuch ſort of Work, but he had a thick Skull.
Now help him for Pity's Sake.

Anſwer. It muſt be cut at one third Part of the Altitude.

Queſt. 13. There is a Ciſtern whoſe Depth is ſeven Tenths of the Width, and the Length is ſix times the Depth, and the ſolid Capacity is 367.5 Feet; I demand the Depth, Width and Length, and how many Buſhels of Corn it will hold.

Firſt, you muſt find three Numbers in Proportion to the Depth, Width and Length, thus; ſuppoſe the Depth 7, then the Width will be 10, and the Length 42; which multiplied together, the Product is 2940, which are the ſolid Inches in a Ciſtern, whoſe Depth is 7, Width 10, and Length 42. But the ſolid Inches in the Queſtion are 635040 ($=367.5\times1728$) then the Cube of the ſuppoſed Width is 1000. So it will be,

As 2940 : 1000 :: 635040 : 216000, whoſe Cube Root is 60, which is the true Width; 7 Tenths thereof is 42, the Depth; and 6 times 42 is 252 Inches, the Length; which three Numbers being multiplied together, the Product will be 635040. If theſe ſolid Inches be divided by 2150.42, the Quotient is 295 $\frac{66610}{215042}$ Buſhels, or 36 Quarters, 7 Buſhels, 1 Peck, 4 Pints. And ſo much will the Ciſtern hold.

Queſt. 14. Suppoſe, Sir, a Buſhel be exactly round,
Whoſe Depth, being meaſur'd, 8 Inches is found,
If the Breadth 18 Inches and half you diſcover,
This Buſhel is legal all *England* over.
But a Workman would make one of another Frame,
Seven Inch and a half muſt be the Depth of the ſame:
Now, Sir, of what Length muſt the Diameter be,
That it may with the former in Meaſure agree?

18.5
18.5

925
1480
185

342.25 the Square.
.7854

136900
171125
273800
239575

268.803150
8

2150.425200 the ſolid Inches in a Buſh.

7.5)2150.4252(286.72336

650
504
542
175
252
270
450

0

.7854)286.72336 . . .
51103
39793
52360
5236

)365.0666(19.107
1
29)265
261
381)406
381
38207)256.600

Anſwer. The Diameter muſt be 19.107 Inches, if the Depth be 7.5 Inches.

Queſt. 15. *In the midſt of a Meadow well ſtored with Graſs,*
I took juſt an Acre to tether my Aſs:
How long muſt the Cord be, that feeding all round,
He mayn't graze leſs nor more than his Acre of Ground?

By *Problem* 10. Section 9. Chap. 1. find the Diameter of a Circle containing an Acre; half that will be the Length of the Cord.

The Work.

660 Feet, the Length of an Acre,
66 Feet, the Breadth of an Acre.

3960
3960

43560 the ſquare Feet in an Acre.

As 1 : 1.2732 : : 43560
43560

763920
63660
38196
50928

55460.5920(235.5 Diamet.
4
117.75 Half.
43)154
129
465)2560
2325
4705) 23559
23525
34

Anſwer. The Cord muſt be 117 Feet and 9 Inches.

But in an *Iriſh* Acre is 70560 Feet, *i. e.* 21 × 21 × 160=70560; then ſay as above,

As 1 : 1.2732 :: 70560
70560
763920
63660
891240
89836.9920(299.72 Diamet.
4
199,86 Half.
49)498
441
589)5736
5301
5987)43599
41509
5994(169020

Queſtion 16. A Malſter has a Kiln that is 16 Feet 6 Inches ſquare; but he is minded to pull it down, and build a new one, that may be big enough to dry three times as much at a time as the old one will do; I demand how much ſquare the new one muſt be?

```
          16.5
          16.5
        ------
         825
        990
      165
      ------
      272.25 the Area of the old one.
           3
      ------
      .  .  .
      816.175(28.57
      4
     -----
  48)416
     384
     ------
 565)3275
      2825
      ------
 5707(45000
      39949
    -------
       5051
       ----
```

Anſwer. The Side of the new one muſt be 28 Feet, and near 7 Inches.

Queſtion 17. If a round Ciſtern be 26.3 Inches Diameter, and 52.5 Inches deep, how many Inches Diameter muſt a Ciſtern be, to hold twice the Quantity, the Depth being the ſame? And how many Ale Gallons will each Ciſtern hold?

```
  26.3
  26.3
  ————
   789
 1578
 526
 ——————
 691.69 the Square.
      2
 ————————
 1383.38(37.19
 9
 ————
67)483
   469
 ——————
741)1438
     741
 ——————
7429)69700
      66861
 ——————————
       2839
```

The Diameter of the greater is 37.19 Inches.

691.69 the Squ. of the leſſer Ciſtern's Diam.
.7854

276676
345845
553352
484183

543.253326 Area of the Baſe.
52.5

2716266630
1086506652
2716266630

28520.7996150 Solid Content in Inches.

282)28520.799(101.137 Gallons.

320
387
1059
2139

165

NOTE, That 282 ſolid Inches is an Ale or Beer Gallon, and 231, a Wine Gallon.

And 359.09 is the Square of the Diameter of a Circle that will hold a Gallon of Ale at an Inch deep, and 294.12 for Wine.

And 217.6 Inches is an *Iriſh* Gallon, of either Ale or Wine. Alſo 277.05 is the Square of the Diameter of a Circle, that will hold an *Iriſh* Gallon at an Inch deep.

You may find the Content in Gallons, thus: Divide the Square of the Diameter by 359.05, and multiply the Quotient by the Depth.

359.05)

359.05)1383.38 . . . (3.853
52.5

306230
18990 19265
1037 7706
19265

The Content of the greater 202.2825 Gallons.

Queſtion 18. If the Diameter of a Caſk at the Bung be 32 Inches, and at the Head 25 Inches, and the Length 40 Inches, how many Ale Gallons are contained therein?

25 32 359
25 32 3

125 64 1077
50 96

625 1024 Square of the Bung Diameter.
1024 the ſame.
625 Square of the Head Diameter.

1077)2673 (2.48
40
5190
8820 99.20

204

But for *Iriſh* Gallons,
Divide by 831.15
The Quotient is 3.21
Which multiply by 40
Give *Iriſh* Gallons 128.40

Anſw. 99.2 Gallons.

Otherwiſe, you may find a mean Diameter, and work by Scale and Compaſſes, thus: Subtract 25 from 32, and there remains 7, which multiplied by .7, the Product is 4.9, which added to 25, the Sum is 29.9. Then extend the Compaſſes from 18.95, the Gage Point *Engliſh*, to 29.9, that Extent turned twice from 40, (the Length) will fall upon 99.6 Gallons ſomething more than before.

Queſtion 19. There is a Stone 20 Inches long, 15 Inches broad, and 8 Inches thick, which weighs 217 Pounds; I demand the Length, Breadth and Thickneſs of another of the ſame Kind and Shape, which weighs 1000 Pounds?

The Cube of 20 (the Length) is 8000. Then, (by *Euclid*, 11. 33.)

As 217 : 8000 :: 1000 : 36870.645, whoſe Cube Root is 33.28 Inches, the Length of the Stone weighing 1000 Pounds. Then ſay,

As 20 : 33.28 :: 15 : 24.96
As 20 : 33.28 :: 8 : 13.312

$$\textit{Anſwer.}\left\{\begin{array}{ll}\text{The Length} & 33.28\\ \text{The Breadth} & 24.96\\ \text{The Thickneſs} & 13.312\end{array}\right\}\text{Inches.}$$

Queſtion 20. If an Iron Bullet, whoſe Diameter is 4 Inches, weighs 9 Pounds, what will be the Weight of another Bullet (of the ſame Metal) whoſe Diameter is 9 Inches?

The Cube of 4 is 64, and the Cube of 9 is 729, Then, (by *Eucl.* 12. 18)

lb. lb.
As 64 : 9 :: 729 : 102.515
lb. *oz.* *dr.*
Anſwer, It weighs 102 8 4 *ferè*.

Queſtion 21. There is a ſquare Pyramid of Marble, each Side of its Baſe is 5 Inches, and the Height thereof 15 Inches, and its Weight is 12 Pounds and a Quarter; I demand the Weight of another like ſquare Pyramid, each Side of whoſe Baſe is 30 Inches?

The Cube of 5 is 125, and the Cube of 30 is 27000. Then, (by *Eucl.* 12. 12.)

lb. lb.
As 125 : 12.25 :: 270002;646
Anſwer. The Weight is 2646 Pounds.

Queſtion 22. There is a Ball or Globe of Marble, whoſe Diameter is 6 Inches, and its Weight 11 Pounds; what will be the Diameter of another Globe of the ſame Marble, that weighs 500 Pounds?

The Cube of 6 is 216. Then,

```
     lb         lb
As 11 : 216 :: 500 : 9818.1818
```

whoſe Cube Root is 21.4 Inches, the Diameter ſought.

Queſtion 23. There is a Frnſtum of a Pyramid, whoſe Baſes are regular Octagons; each Side of the greater Baſe is 21 Inches, and each Side of the leſſer Baſe is 9 Inches, and its Length is 15 Feet; I demand how many ſolid Feet are contained therein?

```
        4.8284 the tabular Number, Page 95.
           237 the Square of a mean Side.
        ------
        337988
       144852              21          12
       96568                9          12
       ---------           ---         ---
       1144.3308           189       3)144
              15            48         ---
       ---------           ---          48
       57216540            237
       11443308
       ---------
144)17164.9620(119.2
    ---------
      276
       1324
         289
          1
```

Anſwer. 119.2 ſolid Feet.

Quſtion 24. There is a Fruſtum of a Cone, the Diameter of the greater Baſe is 36 Inches, and the Diameter of the leſſer Baſe is 20 Inches, and the Length or Height is 215 Inches; I demand the Length and ſolid Content of the whole Cone, and alſo the ſolid Content of the given Fruſtum?

Firſt find the Length of the whole Cone, thus:

From 36
Subtr. 20

As 16 : 215 : : 36 : 483.75
ſo the Length of the whole Cone is 483¾ Inches.

Then find the Content of the whole Cone.

36
36
216
108
1296
.7854
5184
6480
10368
9072
Area Baſe 1017.8784

1017.8784
52.161
10178784
6107270
101788
20357
5089
Feet.
1728)164132.88(94.98
8612
17008
14568
744

Thus I find the Solidity of the whole Cone 94.98 Feet.

Then find the ſolid Content of the top Part that is wanting.

.7854

.7854 the Area of Unity.
400 the Square of 20.

3)314.1600 Area of the leſſer Baſe.

104.72 A third Part.
268.75 Altitude of the top Part.

52360
73304
83776
62832
20944

1728)28143.500(16.28 Feet.

10863
4955
14990

1166

	Feet.
Content of the whole	94.98
Content of the top Piece	16.28
Content of the Fruſtum	78.7

Queſtion 25. If the top Part of a Cone contains 26171 ſolid Inches, and 200 Inches its Length, and the lower Fruſtum thereof contains 159610 ſolid Inches; I demand the Length of the whole Cone, and the Diameter of each Baſe?

200	159610 } add
200	26171 }
40000	185781 The Sum.
200	
8000000	

As 26171 : 8000000 :: 185781 : to 56789881, whoſe Cube Root is 384.3 Inches, the Length of the whole Cone.

Then find the Diameter of the leſſer Baſe, thus;

200)26171
--------
130.855
3
--------
392.565 Area of the leſſer Baſe.

Then, by Prob. 10. Sect. IX. Chap. I.

As 1 : 1.2732 :: 392.565
1.2732
--------
785130
1177695
2747955
785130
392565
--------
. . .
499.8137580(22.35
4
----
42)99
84
----
443)1581
1329
----
4 65)25237
22325
----
2912

Again, As 200	: 22.35	:: 384.3	: 42.94
Leſſer Leng.	Leſſ. Diam.	Great. Leng.	Gr. Diam.
			Inches.

		Inches.
Anſw.	The Length of the whole Cone	384.3
	The Diameter of the greater Baſe	42.94
	The Diameter of the leſſer Baſe	22.35

Queſtion 26. There is a Fruſtum of a Cone whoſe ſolid Content is 20 Feet, and its Length 12 Feet; and the greater Diameter bears ſuch Proportion to the leſſer as 5 to 2; I demand the Diameters.

5×5=25
2×2= 4
5×2=10

The Sum 39

3)12

4)20(5 Feet.

Theſe 5 Feet are the Triple of a mean Area.

Then, as 1 : 1.27324 :: 5 : 6.3662.

So the triple Square of a mean Diameter is 6.3662.

Then, as 39 : 6.3662 :: 25 : 4.080897.

This 4.080897 is the Square of the greater Diameter, whoſe ſquare Root is 2.020123 Feet, which is 24.24147 Inches

Then, as 5 : 24.24147 :: 2 : 9.69659.

So the greater Diameter is 24.24147. and the leſſer Diameter is 9.69659 Inches.

Queſt. 27. There is a Room of Wainſcot 129 Feet 6 Inches in Circumference, and 16 Feet 9 Inches high, (being girt over the Mouldings) there are two Windows, each 7 Feet 3 Inches high, and the Breadth of each from Cheek to Cheek, 5 Feet 6 Inches; the Breadth of the Shutters of each is 4 Feet 6 Inches: the Cheek-Boards and Top and bottom Boards of each Window taken together, is 24 Feet 6 Inches, and their Breadth 1 Foot 9 Inches; the Door-Caſe 7 Feet high, and 3 Feet 6 Inches wide; the Door 3 Feet 3 Inches wide; I demand how many Yards of Wainſcot are contained in that Room?

F.	I.		
129	6		
16	9		
782	0		
129			
64	9		
32	4	6	
2169	1	6	

F.	I.		
3	3		
7	0		
22	9		
11	4	6	Half.
34	1	6	

F.	I.	
7	3	
5	6	
36	3	
3	7	6
39	10	6
		2
79	9	0

F.	I.		
7	3		
4	6		
29	0		
3	7	6	
32	7	6	
16	3	9	Half.
48	11	3	
		2	
97	10	6	

F.	I.	
24	6	
1	9	
24	6	
18	4	6
42	10	6
		2
85	9	0

F.	I.	
3	6	
7	0	
24	6	add.
79	9	
104	3	

The Content of the Room	2169	1	6
The Shutters, at Work and half	97	10	6
The Door, at Work and half	34	1	6
The Cheek-Boards, *&c.*	85	9	0
The Sum	2386	10	6
The Window-lights and Door-caſe deduct	104	3	0
	9)2282	7	6
Anſwer. 253 Yards, 5 Feet.	253	5	

Queſt. 28. There is a Wall which contains 18225 Cube-Feet, and the Height is 5 times the Breadth, and the Length 8 times the Height; what is the Length, Breadth and Height?

Suppoſe the Breadth 2, then the Height muſt be 10, and the Length 80; which three Numbers multiplied together, the Product will be 1600, and the Cube of 2 is 8; then ſay,

As 1600 : 8 : : 18225 : 91.125.

Then the Cube Root of 91.125 is 4.5, which is the Breadth; then 5 times 4.5 is 22.5, the Height; and 8 times 22.5 is 180, the Length.

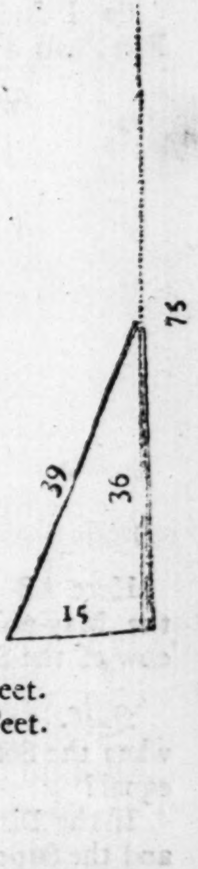
Queſt. 29. There is a May-pole, whoſe Top-End was broken off by a Blaſt of Wind, and the Top-End in falling ſtruck the Ground at 15 Feet diſtance from the Foot of the May-pole; the broken Piece was 39 Feet; now I demand the Length of the May-pole.

By *Eucl.* 1. 47. the Square of the Hypothenuſe of a right-angled Triangle, is equal to the Sum of the Squares of the Baſe and Perpendicular.

Therefore, from the Square of 39 ſubtract the Square of 15, the Square Root of the Remainder is the Piece ſtanding, to which add the Piece broken off, and you have the whole Length.

39

```
   39      15
   39      15
  ———     ———
  351      75
 117      15
  ———     ———
 1521     225
  225
  ———
  . .
 1296(36
   9
  ———
66)396
   396
  ———
  . . .
```



The Piece ſtanding is 36 Feet.
The Piece broken off is 39 Feet.

The whole Length 75

Queſt. 30.

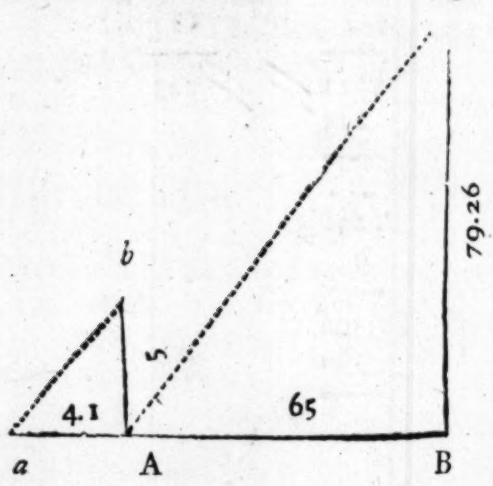
A May-pole there was, whoſe Height I would know;
The Sun ſhining clear, ſtrait to work I did go:
The Length of the Shadow, upon level Ground,
Juſt ſixty-five Feet, when meaſur'd I found:
A Staff I had there, juſt five Feet in Length;
The Length of its Shadow was four Feet one Tenth:
How high was the May-pole, I gladly would know?
And it is the Thing you're deſir'd to ſhew.

By *Eucl.* 6. 4.

As aA : aAb : : AB : BC.

That is,
As 4.1 : 5 :: 65 : 79.26.

So I find the Height of the May-pole to be 79 Feet, and a little above three Inches.



Here AB repreſents the Length of the Shadow of the May-pole, and BC the May-pole; *a*A the Shadow of the Staff, and A*b* the Staff.

Queſt. 31. What will be the Diameter of a Globe, when the Solidity and ſuperficial Content thereof are equal?

If the Diameter be 1, the Solidity will be .5236, and the Superficies will be 3.1416; that is, as 1 to 6. And to find the ſuperficial Content, we muſt multiply 3.1416 by the Square of the Axis or Diameter, and the Product is the ſuperficial Content. And for the Solidity, multiply .5236 by the Cube of the Axis, the Product is the ſolid Content; Therefore, becauſe .5236 is a ſixth Part of 3.1416, we muſt take ſix for the Diameter ſought. For if 3.1416 be multiplied by the Square of 6, *viz.* by 36, the Product will be 113.0976; and if .5236 be multiplied by the Cube of 6, *viz.* by 216, the Product is likewiſe 113.0976, the Solidity equal to the Superficies.

Therefore, 6 is the true Anſwer.

Queſt. 32. What will the Axis of a Globe be, when the Solidity is in Proportion to the Superficies, as 18 to 8?

Becauſe the Solidity and Superficies is as 1 to 6; when the Axis of the Globe is 1, it will be.

As 8 : 18 :: 6 : 13.5.

So the Diameter ſought is 13 $\frac{1}{2}$

If the Proportion of the Solidity to the Superficies had been as 8 to 18, then it will be

As 18 : 8 :: 6 : 2 $\frac{2}{3}$.

So then the Diameter will be 2 $\frac{2}{3}$.

The Reaſon of theſe Operations, both in this and the laſt Queſtion is from Algebra.

Queſt. 33. There are three Grenado Shells of ſuch Capacity, that the ſecond Shell will juſt lie in the Concavity of the firſt, and the third in the Concavity of the ſecond. The Solidity of the Metal of the firſt Shell is equal to its Concavity; and the Solidity of the Metal of the ſecond, to the Concavity, is as 7 to 5; and the Solidity of the third or leaſt Shell's Metal, to its Concavity, is as 9 to 4. Now ſuppoſing the Diameter of the firſt, or greateſt Shell, to be 16 Inches, and allowing every ſolid Inch of Iron to weigh four Ounces; I demand the Diameter of the two leſſer Shells, and the Thickneſs and Solidity of Metal of every Shell, and alſo the Weight of every Shell.

The Cube of 16 is 4096: Then,

As 1 : .5236 :: 4096 : 2144.6656.

The Half thereof is 1072.3328, which is the Solidity of the Metal of the greater Shell, as alſo of the Concavity.

As .5236 : 1 :: 1072.3328 : 2048.

The Cube Root of 2048 is 12.699, which is the Diameter of the ſecond Shell.

The Sum of 7 and 5 is 12; then,

As 12 : 5 :: 1072.3328 : 446.805.

This 446.805 is the ſolid Content of the Concavity of the ſecond.

As .5236 : 1 :: 446.805 : 853.333.

The Cube Root of 853.333 is 9.485, the Diameter of the leaſt Shell.

The Sum of 9 and 4 is 13; then,

As 13 : 4 :: 446.805 : 137.47846.

This 137.47846 is the ſolid Content of the Concavity of the third.

As .5236 : 1 :: 137.47846 : 262.5639.

The Cube Root of 262.5639 is 6.4034, the Diameter of the leaſt Shell's Concavity.

From 16. The Diameter of the greateſt.
Subtr. 12.699 The Diameter of the ſecond.

Rem. 3.301

Half is=1.65 The Thickneſs of Metal of the great.

From 12.699 The Diameter of the ſecond.
Subtr. 9.485 The Diameter of the leaſt.

Rem. 3.214

Half is=1.607 The Thickneſs of Metal of the ſecond.
From 9.485 The Diameter of the leaſt.
Subtr. 6.403 The Diameter of the Concavity.

Rem. 3.082

Half is=1.541 The Thickneſs of Metal of the leaſt.

The Metal of the greateſt is 1072.33 ſolid Inches: which divide by 4, (becauſe every ſolid Inch is a Quarter of a Pound) the Quotient is 268.08 Pounds.

The Metal of the ſecond is 625.52 ſolid Inches; which divided by 4, the Quotient is 156.38 Pounds, the Weight of the ſecond.

The Metal of the leaſt Shell is 309.32 ſolid Inches; which divided by 4, the Quotient is 77.33 Pounds, the Weight of the leaſt.

The Diameter of the	ſecond Shell	12.699	Inches.
	leaſt Shell —	9.485	

The Thickneſs of Metal of the	greateſt —	1.65	Inches.
	ſecond —	1.607	
	leaſt —	1.541	

The Weight of the	greateſt —	268.08	Pounds.
	ſecond —	156.38	
	leaſt —	77.33	

Additional Queſtions in SUPERFICIAL MEASURING.

Queſt. 1. WHAT Difference is there between a Floor, 28 Feet long, by 20 broad, and two others, that meaſure 14 Feet a piece by 10; and what do all three come to, at 45*s. per* Square, *viz.* 10 Feet by 10?

Anſwer, 280 Sq. Ft. Diff. Amount 18*l.* 18*s.*

Queſt. 2. If my Court-yard be 47 Feet 7 Inches ſquare, and I have laid a Foot-way of *Purbec* Stone, 4 Feet wide, along one Side of it: What will paving the reſt, with Flints, come to, at 6*d.* per Yard ſquare?

Anſwer, 5*l.* 15*s.* 2*d.* $\frac{1}{2}$.

Queſt. 3. An Elm Plank is 14 Feet 3 Inches long, and I would juſt have a Yard ſquare ſlit off: At what Diſtance from the Edge, muſt the Line be ſtruck?

Anſwer, 7 Inches $\frac{99}{171}$.

Queſt. 4. I have a wooden Trough, that at 6*d.* per Yard, coſt me 3*s.* 2*d.* Painting within; the Length of it was 102 Inches, the Depth 21 Inches: What was its Breadth?

Anſwer, 2 Feet, 3 $\frac{1}{4}$ Inches.

Queſt. 5. Having a Fiſh Pond of a triangular Form; whoſe three Sides meaſure 400 Yards, 348, and 312: What Quantity of Ground does it cover?

Anſwer, 52284 $\frac{1}{2}$ ſquare Yards.

Queſt. 6. The Height of an Elm, growing in the middle of a circular Iſland, 30 Feet in Diameter, plumbs 53 Feet, and a Line ſtretched from the Top of the Tree, ſtreight to the hither Edge of the Water, 112 Feet: What then is the Breadth of the Moat, ſuppoſing the Land on either Side the Water to be level?

Anſwer, $83\frac{2}{3}$ Feet.

Queſt. 7. Suppoſe a Light-houſe built on the Top of a Rock; the Diſtance between the Place of Obſervation, and that Part of the Rock level with the Eye, and directly under the Building, is given 310 Fathoms; the Diſtance from the Top of the Rock, to the Place of Obſervation, is 423 Fathoms; and from the Top of the Building 425: The Height of the Edifice is required?

Anſwer, 17 Feet, 7 Inches, nearly.

Queſt. 8 A Ladder 40 Feet long, may be ſo planted, that it ſhall reach a Window 33 Feet from the Ground, on one Side the Street; and without moving it at the Foot, will do the ſame by a Window 21 Feet high, on the other Side: The Breadth of the Street is required?

Anſwer, $56\frac{6}{10}$ Feet.

Queſt. 9. The paving of a triangular Court, at 13d per Foot, came to 100l. the longeſt of the three Sides was 88 Feet: What then was the Sum of the other two equal Sides?

Anſwer, 106.85 Feet.

Queſt. 10. There are two Columns in the Ruins of *Perſepolis*, left ſtanding upright; one is 64 Feet above the Plane, the other 50: Between theſe, in a right Line, ſtands an antient Statue, the Head whereof is 97 Feet from the Summit of the higher, and 86 Feet from the Top of the lower Column; the Baſe whereof meaſures juſt 76 Feet to the Center of the Fi-

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gure's Baſe: By theſe Notices, the Diſtance of the Top of the Columns may be, by Numbers, eaſily found?

Anſwer, 157 Feet, nearly.

Queſt. 11. The Surveying Wheel is ſo contrived, as to turn juſt twice in the Length of a Pole, or 16½ Feet: What then is its Diameter?

Anſwer, 2.626 Feet.

Queſt. 12. I would turf a round Plot, meaſuring 130 Feet about, and would know the Charge at 4d. per Yard ſquare?

Anſwer, 2l. 9s. 10d. nearly.

Queſt. 13. It is obſerved, that the extreme End of the Minute-hand of a public Dial, moves juſt 5 Inches in the Space of 3¼ Minutes: The Queſtion is, what is the Length of that Index?

Anſwer, 14.69 Inches.

Queſt. 14. A Pipe of ſix Inches bore will be 3 Hours in running off a certain Quantity of Water: What Time will 4 Pipes, each 3 Inches bore, be in diſcharging double the Quantity?

Anſwer, 6 Hours.

Queſt. 15. A Yard of Rope 9 Inches round weighs, ſuppoſe, 22 lb: what will a Fathom of that weigh, which meaſures a Foot about?

Anſwer, 78 $\frac{2}{9}$ lb.

Queſt. 16. If 20 Feet of Iron Railing ſhall weigh half a Ton, when the Bars are an Inch and Quarter ſquare, what will 50 Feet of Ditto come to at 3½d. per Pound, the Bars being but ⅞ of an Inch ſquare?

Anſwer, 20*l.*

Queſt. 17. A Looking-Glaſs is 16 Inches by 9, and contains a Foot of Glaſs: What will the Content of the Plate be that has twice the Length, and three times the Breadth?

Anſwer, 6 ſquare Feet.

Queſtions in SOLID MEASURING.

Queſt. 1. A Priſm of two equal Baſes, and ſix equal Sides, that meaſures 28 Inches croſs the Center, from Corner to Corner: The ſuperficial and the ſolid Content is required, taking the Length at 134 Inches?

Anſwer, Superf. 9 Yards, 4 Feet, 3 Inches. Solid 39 Feet, 843 Inches.

Queſtion 2. I have a rolling Stone 44 Inches in Circumference, and am to cut off three cubic Feet from one End: Whereabouts muſt the Section be made?

Anſwer, At 33.66 Inches.

Queſt. 3. Suppoſing the Atmoſphere, or Body of the Air and Vapours, ſurrounds the Globe of the Earth and Sea, to 60 Miles above the Surface; the Earth is 7970 Miles in Diameter; how many cubic Yards of Air then hang about, and revolve along with this Planet; and what is the Weight of the whole Maſs of fluid Matter in the Atmoſphere contained, if at a Medium 12 lb *Avoirdupois*, be found experimentally, as in Fact it is, to preſs upon every circular Inch, on the Surface of the Earth?

Anſwer, Meaſure 66264254274148761600 ſolid Yards, Weight 1224019163676672000 Pounds *Avoirdupois*.

Queſt. 4. A Cork may be cut into ſuch a Form, that it may, without Alteration, ſeverally fill the Cavity of a Circle an Inch in Diameter, of an equilateral Triangle, whoſe Sides are each an Inch, and a geometrical Square, alſo an Inch, on a Side: What muſt be the Shape?

Anſwer, It will be a Wedge, whoſe Baſe, being circular, will fill the Round: The Ridge-Front will repleniſh the Square, and the End-Section make out the Triangle.

Queſt. 5. Let it be demanded to find the Side and ſolid Content of the greateſt Cube that can be contained in a Globe of 27 Inches Diameter.

Queſt. 6. If a Piece of Timber 48 Feet long, 6 Inches broad, and 14 Inches thick, coſt 10*l.* what's the Value of 5 ſuch Pieces which are 8 Feet long, 4 Inches broad, and 6 Inches thick?

Queſt. 7. If 300 Men in 15 Days dig a Trench 5600 Feet long, 6 Feet deep, and 12 wide; how long muſt that Trench be whoſe Depth is 8 Feet, and Width 14 Feet, dug by 2700 Men in 25 Days?

Queſt. 8. If 24 Men working 189 Days, 12 Hours each Day, dig a Trench $33\frac{1}{2}$ Yards long, $3\frac{1}{2}$ deep, and $5\frac{2}{7}$ wide; how many Hours per Day muſt 217 Men work to dig a Trench 23 Yards long, $2\frac{1}{2}$ deep, and $3\frac{1}{3}$ wide in 6 Days?

Queſt. 9. A Bath-ſtone, 20 Inches long, 15 over, and 8 deep, weighs 220 lb. How many cubic Feet thereof will freight a Ship of 290 Ton?

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A SHORT
APPENDIX.

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§ I. *Of* GAGING, *according to both the* Iriſh *and* Engliſh *Gallon.*

I SHALL not here give the whole Art of Gaging, (there being ſeveral Books of that Art already in Print, wrote by better Hands) but ſhall only lay down ſome ſhort practical Rules, whereby any Artificer, or others, may find the Quantity of Liquor in any Veſſel, upon Occaſion.

PROBLEM I.

To find the ſeveral Multipliers, Diviſors and Gage-points, belonging to the ſeveral Meaſures now uſed in England *and* Ireland.

282)1.0000(.003546 Multiplier for Ale Gallons.
231)1.0000(.004329 Multiplier for Wine Gallons.
268.8)1.000(.0037202 Multiplier for Corn Gallons.
2150.42)1.000.(.00046502 Multiplier for Corn Buſhels.
217.6)1.0000(.0045955 Multiplier for *Iriſh* Gallons.

So, if the ſolid Inches in any Veſſel be multiplied by the ſaid Multipliers, the Product will be Gallons in the reſpective Meaſures: Or, dividing by the Diviſors 282, 231, 268.8, or 217.6, the Quotients will likewiſe be Gallons.

NOTE, That 282 ſolid Inches is a Gallon of Ale or Beer-Meaſure; 231 ſolid Inches is a Gallon of Wine-Meaſure; 268.8 ſolid Inches is a Gallon, and 2150.42 ſolid Inches is a Buſhel of Corn-Meaſure. Alſo 217.6 ſolid Inches is a Gallon *Iriſh*, both of Ale, Wine, or Oyl.

For circular Areas, the following Multipliers and Diviſors are to be uſed.

282).785398(.002785 Multiplier for Ale Gallons.
231).785398(.003399 Multiplier for Wine Gallons.
217.6).785398(.0036093 Multiplier for *Iriſh* Gal.
.785398)282(359.05 Diviſor for Ale Gallons.
.785398)231(294.12 Diviſor for Wine Gallons.
.785398)2150.42(2738 Diviſor for Corn Buſhels.
.785398)217.6(277.05 Diviſor for *Iriſh* Gallons.
The Square-Root of the Diviſor is the Gage-point.

The Gage point for Squares in	Ale Meaſure,	is	16.79
	Wine Meaſure,		15.19
	Malt-Buſhel,		46.36
	Iriſh Gallons,		14.75

The Gage-point for circular Figures in	Ale-Meaſure,	is	18.95
	Wine-Meaſure,		17.15
	Malt-Buſhel,		52.32
	Iriſh Gallon,		16.64

PROBLEM II.

To find the Area in Ale, Wine, or Iriſh *Gallons, of any rectilineal plane Figure, whether Triangular, Quadrangular, or Multangular.*

TO reſolve this *Problem*, you muſt, by Chap. I. Part II. find the Area in Inches, and then bring it into Gallons, by dividing that Area in Inches by the proper Diviſor, *viz.* by 282 for Ale, or by 231 for Wine, or 217.6 for *Iriſh*; or elſe by Multiplication, by .003546 for Ale, by .004329 for Wine, by .0045955 for *Iriſh*, and the Quotient or Product will be the Area.

Example. Suppoſe a Back or Cooler in the Form of a Parallelogram, or Long Square, 250 Inches in Length, and 84.5, Inches in Breadth: what is the Area in Ale, Wine, or *Iriſh* Gallons.

Multiply 250 by 84.5, and the Product is 21125, the Area in Inches; which divided by 282, and the Quotient is 74.9 Gallons of Ale; or multiplied by .003546, the Product is 74.90925 Gallons, nearly the ſame; and if 21125 be divided by 231, or multiplied by .004329, it will give 91.45 Gallons of Wine; and if 21125 be divided by 217.6, or multiplied by .004595, it will give 97.06 *Iriſh* Gallons.

By Scale *and* Compaſſes.

Extend the Compaſſes from 282 to 250, that Extent will reach from 84.5 to 74.9: And,

Extend from 231 to 250, that Extent will reach from 84.5 to 91.45.

Extend the Compaſſes from 217.6 to 250, that Extent will reach from 84.5 to 97.06.

NOTE, the Areas of all Superficies are always to be underſtood to be one Inch deep, otherwiſe it could not be ſaid that the Area of ſuch a Parallelogram, Circle, *&c.* is ſo many Gallons.

Having found the Area of a Back or Cooler, the next Thing will be to find out the true Dipping or Gaging-place in that Back, that ſo the true Quantity of Worts may be computed at any Depth; which may be thus done.

1. When the Bottom of the Back is covered all over (of any Depth) with Worts or other Liquor, then dip it in eight or ten ſeveral Places, (more or leſs, according to the Largeneſs of the Back) as remote and equally diſtant from each other as you can well do, noting down the wet Inches and Decimal Parts of every Dip.

2. Divide the Sum of all thoſe Dips by the Number of Places you dipped in, and the Quotient will be the mean wet of all thoſe Dips.

3. *Laſtly*, find out ſuch a Place by the Side of the Back (if you can) that juſt wets the ſame with that mean Dip, and make a Notch or Mark there for the true and conſtant Dipping-place of that Back.

Then, if any Quantity of Worts (which covers the whole Back) be dipped or gaged at that Place, and the wet Inches ſo taken be multiplied into the Area of the Back in Gallons, the Product will ſhew how many Gallons of Wort are in that Back at that Time, provided the Sides of the Back do ſtand at Right-angles with the Bottom.

PROB-

PROBLEM III.

The Diameter of a Circle being given in Inches, to find the Area thereof in Ale, Wine, or Iriſh *Gallons.*

IF the Square of the Diameter be multiplied by .002785 for Ale, or by .003399 for Wine, or by .0036093 for Iriſh Gallons; or if it be divided by 359.05 for Ale, or by 294.12 for Wine, or by 277.05 for Iriſh Gallons, the Products or Quotients will be the reſpective Ale, Wine, or Iriſh Gallons.

Example. Suppoſe the Diameter of a Circle be 32.6 Inches, what will be the Area in Ale, Wine, or Iriſh Gallons?

The Square of 32.6 is 1062.76.

Then 359.05) 1062.76(2.9599 Area in Ale Gal.
And 294.12) 1062.76(3.6133 Area in Wine Gal.
And 277.05) 1062.76(3.8565 Area in Iriſh Gal.
Or 1062.76×.002785=2.9599 Ale Gal.
And 1062.76×.003399=3.6133 Wine Gal.
And 1062.76×.003609=3.8565 Iriſh Gal.

By Scale *and* Compaſſes.

Extend the Compaſſes from 18.95 (the Gage Point for Ale) to 32.6 (the Diameter) that Extent will reach from 1 to a fourth Number, and from that fourth to 2.9599 Gallons. Or, extend the Compaſſes from 1 to 32.6, that Extent, turned twice over from .002785 will at laſt fall upon 2.9599.

For Wine: Extend from 17.15 (the Gage-Point for Wine) to 32.6 (the Diameter) that Extent turned twice over from 1, will at laſt fall upon 3,6133 Gallons.

Or thus: Extend from 1 to 32.6, that Extent will reach from .003399 (being twice turned over) to 3.6133 Wine Gallons.

For Iriſh Gallons: Extend from 16.64 (the Iriſh Gage-Point) to 32.6 (the Diameter) that Extent, turned twice over from 1, will at laſt fall upon 3.8565, the Iriſh Gallons ſought.

PROBLEM IV.

The Tranſverſe (or longeſt Diameter) and the Conjugate (or ſhorteſt Diameter of an Ellipſis (or Oval) being given, to find its Area in Ale, Wine, or Iriſh Gallons.

IF the Rectangle, or Product of the two Diameters, that is, of the Length and Breadth of the Oval, be divided by 359.05, or multiplied by .002785 for Ale, or divided by 294.12, or multiplied by .003399 for Wine, or divided by 277.05, or multiplied by .0036093 the Quotients or Products will be the Ale, Wine or Iriſh Gallons required.

Example. Suppoſe the longeſt Diameter be 81.4 Inches, and the ſhorteſt Diameter be 54.6 Inches, what will be the Area of that Oval?

Multiply 81.4 by 54.6, and the Product is 4444.44; then

359.05)4444.44(12.38 Area in Ale Gal.
294.12)4444.44(15.11 Area in Wine Gal.
277.05)4444.44(16.71 Area in *Iriſh* Gal.
Or 4444.44×.002785=12.38 Ale Gal.
And 4444.44×.003399=15.11 Wine Gal.
And 4444.44×.003609=16.1 *Iriſh* Gal.

By

By Scale *and* Compaſſes.

Firſt find a mean Proportional between 81.4 and 54.6, by dividing the Diſtance between them into two equal Parts, and the middle Point will be at 66.6, which is the mean Proportional, that is, the Diameter of a Circle equal to the Oval. Then extend the Compaſſes from 18.95 (the Gage-point for Ale) to 66.6, that Extent turned twice over from 1, will at laſt fall upon 12.38 Ale-Gallons: And extended from 17.15 (the Gage-point for Wine) to 66.6, that Extent, turned twice over from 1, will reach at laſt to 15.11 Wine Gallons.

Laſtly, Extend the Compaſſes from 16.64 (the *Iriſh* Gage-point) to 66.6, that Extent, turned twice over from 1, will at laſt fall upon 16.1 *Iriſh* Gallons.

PROBLEM V.

To find the Content in Ale, Wine, or Iriſh *Gallons, of any Priſm, what Form ſoever its Baſe is of.*

FIRST find its ſolid Content in Inches, (by Sect. 1, 2, 3, of Chap. II. Part II.) then divide that Content in Inches by 282 for Ale, or by 231 for Wine, or by 217.6 for *Iriſh*; the reſpective Quotients will be the Content in Wine, Ale, or *Iriſh* Gallons.

Otherwiſe, you may find the Content of a Priſm, by finding the Area of its Baſe in Gallons, (by *Problem* II. of this Appendix) and multiply that Area by the Tun's Height or Depth within, the Product will be its Content in Gallons.

Example. Suppoſe a Tun, whoſe Baſe is a Parallelogram right-angled, its Length being 49.3 Inches, its Breadth 36.5 Inches, and the Depth of the Tun is 42.6 Inches; the Content in Ale, Wine, and *Iriſh* Gallons is required.

The Length, Breadth and Depth being multiplied continually, the Product is 76656.57; which divided by 282, the Quotient is 271.83 Ale Gallons; and divided by 231, the Quotient is 331.84 Wine Gallons; and divided by 217.6, the Quotient is 252.23 *Iriſh* Gallons; and by dividing by 2150.4, ſuch a Ciſtern will be found to hold 35.65 Buſhels of Corn.

By Scale *and* Compaſſes.

Extend the Compaſſes from 282 to 36.5, (the Breadth of the Baſe, that Extent will reach from 49.3 (its Length,) to 6.38 Ale Gallons, the Area of the Baſe) Then extend from 1 to 42.6, (the Depth) that Extent will reach from 6.38 (the Area of the Baſe) to 271.8 Gallons, the Content.

PROBLEM VI.

To find the Content of a Tun, whoſe Baſes are alike and parallel, but unequal, being the Fruſtum of a Pyramid.

FIND the Area of each Baſe, and a mean Proportional between them, and multiply the Sum of thoſe three by one third Part of the Depth or Height, and the Product is the Content.

Example. Suppoſe a Tun whoſe Baſes are Parallelograms, the Length of the greater is 100 Inches, and its Breadth 70 Inches; the Length of the leſſer Baſe 80, and its Breadth 56; and the Depth of the Tun 42 Inches; the Content in Ale and Wine Gallons is required.

Multiply 100 by 70, the Product is 7000, the Area of the greater Baſe; and 80 multiplied by 56, the Product is 4480, the Area of the leſſer Baſe; then multiply the two Areas into each other, and the Product is 31360000, whoſe ſquare Root is 5600, a geometrical mean Proportional.

The greater Area	7000	add
The leſſer Area	4480	
The mean Proportional	5600	
	17080	
A third of the Depth	14	
	68320	
	17080	

282)239120(847.94 Ale Gal.
231)239120(1035.15 Wine Gal.
217.6)239120(1098.9 Iriſh Gal.

PROBLEM VII.

To find the Content of a Tun, whoſe Baſes are parallel and circular, being the Fruſtum of a Cone.

YOU may find the Content as in the laſt *Problem*, by multiplying the Sum of the Areas of the two Baſes, and a mean Proportional, by one third Part of the Depth.

But it will be a ſhorter Way to find the Area of a mean Circle in Gallons, and multiply that by the Depth, thus: To the Rectangle of the greater and leſſer Diameters, add one-third Part of the Square of the Difference of the Diameters, that Sum is the Square of a mean Diameter, which divided by 359.05 for Ale, or by 294.12 for Wine, or by 277.05 for *Iriſh* Gallons, gives the Area of a mean Circle in Ale, Wine, or *Iriſh* Gallons; which multiplied by the Depth, gives the Content.

Example. Suppoſe the greater Diameter 80 Inches, and the leſſer Diameter 71 Inches, and the Depth 34 Inches; the Content in Ale, Wine, and *Iriſh* Gallons is required.

Multiply 80 by 71, and the Product is 5680; to which add 27, (a third Part of the Square of the Difference of the Diameters) and the Sum is 5708, which is the Square of a mean Diameter; which divide by 359.05, and the Quotient will be 15.895 Ale Gallons the Area; which multiply by 34, (the Depth) and the Product will be 540.43 Gallons the Content.

By Scale *and* Compaſſes.

Add the two Diameters together, and take half the Sum, which is 75.5, which take for a mean Diameter, (tho' it is not exact, yet it will be near enough the Truth, if the Difference between the Diameters be not great) extend the Compaſſes from 18.95, the Gage-point for Ale, to 75.5, the mean Diameter; that Extent will reach from 34, the Depth, to a fourth Number, and from that to 540.4 Gallons the Content.

And if you extend the Compaſſes from 17.15, the Gage-point for Wine, to 75.5, that Extent will reach from 34 (twice turned over) to 659.7 Gallons of Wine.

Laſtly, If you extend the Compaſſes from 16.64, the *Iriſh* Gage-point, to 75.5, that Extent will reach from 34 (twice turned over) to 700.3 *Iriſh* Gallons.

The Method uſed by the Gagers for all ſuch Tuns, is to take the Diameter in the Middle of every 10 Inches, that is, at five Inches from the Bottom, and at 15, and at 25, *&c.*

Then they find the Area to every one of theſe Diameters, and enter them in their Books: Then, when they ſurvey, they take the wet Inches and Parts that the Liquor in the Tun is in Depth, and every 10 Inches they take the reſpective Areas, and remove the ſeparating Point one Place towards the Right Hand, and for what odd Inches of the Depth above the even Tens, they multiply the next Area by them, and ſo add all the ſeveral Products together, and the Total will be the Gallons of Liquor in the Tun.

Example. Suppoſe the Diameter at 5 Inches from the Bottom be 64 Inches, and at 15 Inches from the Bottom 67 Inches, and at 25 Inches 70 Inches, and at 35 Inches from the Bottom, the Diameter is 73 Inches. Now the Area anſwering 64 Inches, is 11.4078 Ale Gallons, and to 67 Inches is 12.5023 Gallons; and the Area to 70 Inches is 13.647 Gallons, and to 73 is 14.8418 Gallons: Then, ſuppoſing the Depth of the Liquor in the ſaid Tun be found to be 33.6 Inches; now, to caſt up this Gage, firſt, in the Area anſwering to 64 Inches, being multiplied by 10, that is, by removing the ſeparating Point a Place towards the Right Hand, it will be 114.078 Gallons, and the next will be 125.023, and the next 136.47 Gallons. Now theſe three will be the Content to 30 Inches deep. Then to find the Content of the 3.6 Inches, multiply the next Area

14.8418

14.8418 by 36, and the Product is 53.4305; add all theſe together, and the Sum is the whole Quantity of Liquor in the Tun.

The Content at 10 Inches deep	114.078
The Content at the next 10 Inches	125.023
The Content of the next 10 Inches	136.470
The Content of the 3.6 Inches	53.430
The whole Quantity of Liquor in the Tun, Ale Gallons,	429.001

PROBLEM VIII.

To find the Drip or Fall of a Tun.

SUPPOSE the Tun laſt mentioned was ſo placed, that when the Bottom is but juſt covered on one Side, the Liquor is 4 Inches deep on the Side oppoſite; how much muſt be allowed for the Fall of this Tun; that is, how much Liquor is there in the Tun?

The Diameter in the Middle of 4 Inches from the Bottom, is 61.6 Inches, and the Area anſwering thereunto is 10.568; which multiplied by 2, (that is half 4) the Product is 21.136 Ale Gallons; and ſo much Liquor will juſt cover the Bottom.

But ſuppoſe it was ſet ſo much on one Side, as to be 30 Inches deep on one Side, when the Liquor on the oppoſite Side juſt cuts between the Bottom and Staves, how much Liquor will there be in the Tun?

Square the Bottom Diameter, and multiply that Square by the top Diameter, and divide the laſt Product by the Sum of the Diameters, and to the

Quotient add the Square of the Bottom Diameter, and divide the Sum by 1077.15 for Ale, or by 882.36 for Wine, or by 831.15 for *Iriſh* Gallons, multiply the Quotient by the Depth, the Product is the Content.

The Bottom Diameter of the forementioned Tun is 61 Inches, and the Diameter at 30 Inches from the Bottom is 71.5 Inches; the Square of 61 is 3721, which multiplied by 71.5, the Product is 266551.5; this divided by 32.5, the Sum of the Diameters, the Quotient is 2007.936: to which add 3721, (the Square of 61) and the Sum will be 5728.936; this divided by 1077.15, the Quotient is 5.3186; which multiplied by 30, the Depth, the Product is 159.558, the Gallons of Liquor in the Tun.

When the Fruſtum of a Cone, or Pyramid, is cut by a Diagonal Plane thro' the Extremities of the Diameters, as the Liquor in the Tun repreſents, ſuch Solid is called a Hoof. (*Vide Ward's* Young Mathematician's Guide, Page 414.)

If it be the Hoof of a ſquare Fruſtum, inſtead of dividing by 1077.15, divide by 846 for Ale, or by 613 for Wine, or by 652.3 for *Iriſh* Gallons. All the reſt of the Work is the ſame.

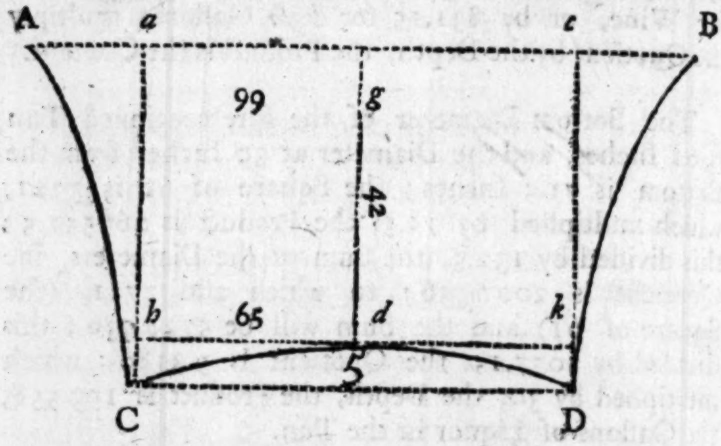
PROBLEM IX.

To Gage a COPPER.

LET ABCD be a ſmall Copper to be gaged.

Take a ſmall Chord, or Packthread, make one End faſt at A, and extend the other to the oppoſite Side of the Copper at B, where make it faſt, or cauſe ſome Perſon to hold it very ſtrait; then ſet one End of the Inſtrument in the Bottom of the Copper at C, and move it to and fro till you find the neareſt Diſtance

tance to the Thread (as at *a*) this Diſtance, *a* C, is the Depth of the Copper, which ſuppoſe to be 47 Inches.



In like manner, ſet the End of the Rule upon the Top of the Crown at *d*, and take the neareſt Diſtance to the Thread, (as *dg*) which ſuppoſe 42 Inches, this ſubtracted from *a* C 47, the Remainder 5 is the Altitude of the Crown.

To find CD, the Diameter of the Bottom of the Crown:

Meaſure AB, the Diameter of the Top, which admit it be 99 Inches, then hold a Thread ſo as a Plummet at the End thereof may hang juſt over C, by which means you will find the Diſtance *a* A. Do the like on the other Side, ſo will you find alſo the Diſtance *c*B, which ſuppoſe 17.5 Inches each; add theſe two together, and ſubtract their Sum, *viz.* 35, from 99, and the Remainder is 64 Inches, the Diameter at the Bottom of the Crown: The Diameter which touches the Top of the Crown may be found by the Sliding Rule to be 65 Inches.

Now to find the Content of the Copper from the Crown upwards, that is, the Part AB*kb*, the Depth *gd* being 42 Inches, you may take a Diameter in the Middle of every 6 Inches of the Depth, which ſuppoſe to be as in the ſecond Column of the following Table, the Numbers in the third Column are the reſpective Areas in Ale Gallons, found by *Problem* III. the fourth Column ſhews the Content of every 6 Inches; all which being added together, the Sum will be the Content of that Part AB*kh*; that is, ſo much as it will hold after the Crown is covered.

Now, if the Crown be taken for the Fruſtum of a Sphere, the Content (by the latter Part of Sect. XI. *Page* 187) will be found to be 28.75 Gallons.

But may be more readily found, very near the Truth, thus:

The Diameter CD was found to be 64, and the Area to this Diameter is 11.408; this multiplied by half the Crown's Altitude, *viz.* by 2.5, gives 28.52 Ale Gallons, the Content of the Crown.

The Content of the Part *bk*DC is 57.935 Ale Gallons, from which ſubtract the Content of the Crown, 28.52, and the Remainder is 29.415 Ale Gallons, and ſo much Liquor will juſt cover the Crown,

Parts of the Depth	Diameters	Areas.	Content of every ſix Inches
6	95.3	25.2945	151.767
6	90.1	22.6095	135.657
6	85.0	20.1223	120.734
6	80.	17.8246	106.947
6	75.2	15.7499	94.499
6	70.5	13.8426	83.056
6	66.	12.1319	72.791
The Sum			765.451
To juſt cover the Crown			29.415
The whole Content in Ale Gallons			794.866

By Scale *and* Compaſſes.

You may find the Areas anſwering to every one of the Diameters, thus

Extend the Compaſſes from the Gage-point to the Diameter, that Extent, being twice turned over from 1, will at laſt fall upon the Area of that Circle; or being twice turned over from 6, will give the Content of that 6 Inches of the Depth.

Example. Extend the Compaſſes from 18.95 (the Gage-point) to 95.3 that Extent, turned twice over from 6, will at laſt fall upon 151.76 Gallons, the Content of the firſt ſix Inches. And ſo of the reſt.

PROBLEM X.

To compute the Content of any cloſe Caſk.

IN order to perform this difficult Part of Gaging, the three following Dimenſions of the Caſk muſt be truly taken,

Viz. The Bung-Diameter,
The Head-Diameter,
The Length of the Caſk, within the Caſk.

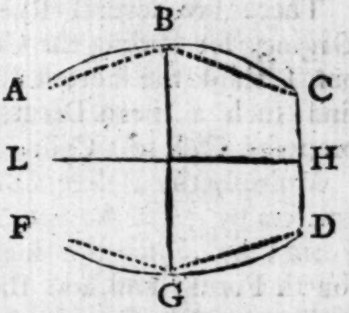
In taken theſe Dimenſions it muſt be carefully obſerved.

1. That the Bung-hole be in the Middle of the Caſk; alſo, that the Bung-ſtaff, and the Staff oppoſite to the Bung-hole, are both regular, and even within.

2. That the Heads of the Caſk are equal, and truly circular; if ſo, the Diſtance between the Inſide of the Chine to the Outſide of its oppoſite Staff, will be the Head Diameter within the Caſk, very near.

3. With a ſliding Pair of *Callipers* (made for that Uſe) take the ſhorteſt Diſtance, or Length, between the Outſides of the two Heads; from that Length ſubtract 1 ½ Inch, more or leſs, according to the Largeneſs of the Caſk, for the Thickneſs of the Head; the Remainder will be the Length of the Caſk within. But if the Caſk be empty, you may take the Length by putting a ſtrait Rod in at the Tap-hole, and allow for the Thickneſs of the Head.

Now, by theſe Dimenſions, one would think the Content of the Caſk was perfectly limited; but it will be eaſy to perceive, by the following Figure, that the Diameters and Length of one Caſk may be equal to thoſe of another, and yet one of thoſe Caſks may contain ſeveral Gallons more than the other.



As for Inſtance, the Figure ABCDF is ſuppoſed to repreſent a Caſk: Then is is plain, that if the outward curved Lines, ABC and FGD, are the Bounds, or Staves of the Caſk, it muſt needs hold more than if the inner pricked Lines where the Bounds or Staves; and yet the Bung-Diameter BG, and Head-Diameters CD and AF, and Length LH, are the ſame in both thoſe Caſks.

Whence it appears, that no one general Rule can be given, whereby the Content of all Sorts of Caſks can be gaged. And therefore Gagers do uſually ſuppoſe every Caſk to be in ſome of theſe Forms.

1. The middle Fruſtum of a Spheriod.
2. The middle Fruſtum of a Parabolick Spindle.
3. The lower Fruſtums of two equal Parabolick Conoids.
4. The lower Fruſtums of two equal Cones.

1. If the Staves of the Caſk be very much curved (as the outward Lines of the laſt Figure) then the Caſk is ſuppoſed to be the middle Fruſtum of a Spheroid.

2. If the Staves between the Bung and the Head be ſomething leſs curved, then the Caſk is taken to be the middle Fruſtum of a Parabolick Spindle.

3. If the Staves (between the Bung and Head) be very little curved, then the Caſk is taken to be the lower Fruſtums of two equal Parabolick Conoids, abutting or joining together, upon one common Baſe.

4. If the Staves (between the Bung and Head) be ſtrait, as the pricked Line in the laſt Figure, then the Caſk is taken to be the lower Fruſtums of two

equal Cones, abutting or joining together upon one common Baſe.

There are ſeveral Rules laid down in Books of Gaging, for finding the Content of each ſeveral Form, but I think the ſhorteſt and moſt practical Way is to find ſuch a mean Diameter, which will reduce the propoſed Caſk to a Cylinder; thus,

Multiply the Difference of the Bung and Head Diameters by .7 for the Spheroid; by .65 for the ſecond Form; by .6 for the third Form, and by .55 for the fourth Form; and add the Product to the Head Diameter, and the Sum is a mean Diameter.

Example. Suppoſe the Bung-Diameter be 32 Inches, the Head-Diameter 24 Inches, and the Length 40 Inches, the Content in each Variety is required.

The Difference between the Bung and Head-Diameters is 8, which multiplied by .7, the Product is 5 6; which added to the Head-Diameter, the Sum is 29.6 for the mean Diameter; the Area anſwering thereunto will be found by *Prob.* III. to be 2.44 Ale Gallons: which multiplied by the Length, the Product is 97.4 Gallons, and ſo much is the Content, if it be of the firſt Form.

Again. If the Difference of the Diameters 8, be multiplied by .65, the Product will be 5.2; which added to the Head-Diameter, the Sum is 29.2 for the mean Diameter; and the Area anſwering thereunto is 2.3746 Gallons; which multiplied by 40, the Length, the Product is 94.98 Gallons, the Content, if it be of the ſecond Form.

Again, If the Difference 8, be multiplied by 6, the Product is 4.8; which added to the Head-Diameter, the Sum is 28.8, the mean Diameter; the Area thereunto is 2.31 Gallons; which multiplied by 40, gives the Content 92.4 Gallons for the third Form.

Again, The Difference 8, multiplied by .55, the Product is 4.4; which added to the Head-Diameter, makes the mean Diameter 28.4; the Area thereof is

2.2463, which multiplied by 40, the Product is 89.85 Gallons for the third Form.

By Scale *and* Compaſſes.

Extend the Compaſſes from the Gage-point 18.95, to the firſt mean Diameter 29.6, that Extent will reach from the Length 40, to a fourth Number, and then to the Content 97.4 Ale Gallons.

Again, Extend from 18.95 to 29.2, (the ſecond mean Diameter) that Extent, turned twice over from 40, will at laſt fall upon 94.48 Gallons.

Again, Extend from 18.95 to 28.8, the third mean Diameter) that Extent, turned twice over from 40, will at laſt fall upon 92.4 Gallons.

Again, Extend from 18.95 to 28.4, (the fourth mean Diameter) that Extent, turned twice over from 40, will at laſt fall upon 89.85 Gallons

Altho' I have all along made Uſe of the Line of Numbers upon the Common two Foot or eighteen Inch Rules, for the Reaſon mentioned in the *Preface*, yet the Rules may eaſily be applied to the Sliding Rule, thus: To find the Area of a Circle in Gallons, ſet the Gage-point upon D, (that is, a ſingle Line of Numbers) to 1 upon C, (that is, a double Line) then againſt any Diameter upon D, is the Area upon C, thus:

To find the Content of the Caſk laſt mentioned. The firſt Form:

Set the Gage-point 18.95 upon D, to the Length 40 upon C; then, againſt the mean Diameter 29.6 upon D, is 97.4 Gallons, the Content upon C.

And againſt 29.2 (the next mean Diameter) on D, is 94.98 Gallons on C.

And againſt 28.8 (the next mean Diameter) on D, is 92.4 Gallons on C.

And againſt 28.4 (the laſt mean Diameter) on D, is 89.85 Gallons on C.

All done without removing the Slider.

A TABLE *of the Segments of a Circle whoſe Area is Unity.*

V. S.	Segm.	V. S,	Segm.	V. S.	Segm.	V. S.	Segm.
1	.0017	99	.9983	26	.2066	74	.7934
2	.0048	98	.9952	27	.2178	73	.7822
3	.0087	97	.9913	28	.2292	72	.7708
4	.0134	96	.9866	29	.2407	71	.7593
5	.0187	95	.9813	30	.2523	70	.7477
6	.0245	94	.9755	31	.2640	69	.7360
7	.0308	93	.9692	32	.2759	68	.7241
8	.0375	92	.9625	33	.2878	67	.7122
9	.0446	91	.9554	34	.2998	66	.7002
10	.0520	90	.9480	35	.3119	65	.6881
11	.0598	89	.9402	36	.3241	64	.6759
12	.0680	88	.9320	37	.3364	63	.6636
13	.0764	87	.9236	38	.3487	62	.6513
14	.0851	86	.9149	39	.3611	61	.6389
15	.0941	85	.9059	40	.3735	60	.6265
16	.1033	84	.8967	41	.3860	59	.6140
17	.1127	83	.8873	42	.3986	58	.6014
18	.1224	82	.8776	43	.4112	57	.5888
19	.1323	81	.8677	44	.4238	56	.5762
20	.1424	80	.8576	45	.4364	55	.5636
21	.1526	79	.8474	46	.4491	54	.5509
22	.1631	78	.8369	47	.4618	53	.5382
23	.1737	77	.8263	48	.4745	52	.5255
24	.1845	76	.8155	49	4873	51	.5127
25	.1955	75	.8045	50	.5000	50	.5000

The Uſe of the Table of Segments.

§ 1. *Is to find the Ullage, or Quantity of Liquor remaining in a Cask, whoſe Axis is parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Cask.*

The RULE *is.*

TO the wet or dry Inches of the Bung-Diameter add a competent Number of Cyphers, then divide

vide it by the whole Diameter, the Quotient found in the Table under the Title V. S. gives a Segment; which multiplied by the whole Content of the Caſk, the Product ſhews the Quantity of Liquor in the Caſk, if the Dividend was the wet Inches, or the Ullage, if it was the dry.

Let there be a Caſk in form of a Cylinder, whoſe Bung-Diameter is 29 Inches, the dry Part 13, and the wet 16, and the Content 80 Gallons; how many Gallons are wanting to fill the Caſk?

Divide the dry Inches 13, by 29 the Bung-Diameter, and the Quotient is .448; find the two firſt Figures .44 under V. S. and the Segment againſt it is .4238, to which add a proportional Part for the 8, and the whole Segment will be .4333; which multiplied by the Content of the Caſk, the Product will be 34.664 Gallons; and ſo much the Caſk wants of being full.

NOTE, If the Caſk be in the Form of a Cylinder, or near that Figure, the Table will give the Ullage exact enough; but if it be a ſpheroid Caſk, then uſe the following Method:

1. By the Bung and Head-Diameters, find ſuch a mean Diameter as you judge will reduce the propoſed Caſk to a Cylinder, and then find its Content.

2. From the Bung-Diameter ſubtract the mean Diameter, and take half the Difference.

3. From the wet Inches ſubtract the ſaid half Difference; reſerve this Difference, then uſe this Proportion;

As the mean Diameter : is to 100,
(the Diameter of the tabular Circle) : :
ſo is the reſerved Difference :
to a verſed Sine in the Table.

Then, if the tabular Segment be multiplied into the Content (as before) the Product will be the Quantity of Liquor in the Caſk.

Example. Let the Caſk be the ſame as in Page 317 of the firſt Form, where the Bung-Diameter is 32 Inches, and the mean Diameter 29.6, and the Content 97.4 Gallons; and ſuppoſe the wet Inches 19, to find the Quantity of Liquor in the Caſk.

From	32	From	19
Subtr.	29.6	Subtr.	1.2
Rem.	2.4	Rem.	17.8 reſerved.
Half	1.2		

As 29.6 : 100 : : 17.8 : 60, the V. S.

The Segment to 60 is .6265, which multiplied by 97.4, the Content, the Product is 61 Gallons, the Quantity of Liquor in the Caſk.

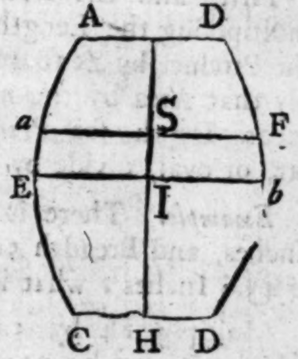
If the dry Inches had been given, by the ſame Method you might have found the Ullage, or what the Caſk wanted of being full.

§ 2. *To find what Quantity of Liquor is in a Caſk, when its Axis is perpendicular to the Horizon,* viz. *when it ſtands upright upon one of his Heads.*

TO do this, you muſt know how to calculate the Area of any Circle, between the Bung and Head, whoſe Diſtance from the Bung, or Middle of the Caſk is given; which may be done by this Proproportion:

As the Square of half the Length of the Caſk : is to the Difference between the Bung and Head-Areas : : ſo is the Square of any Circle's Diſtance from the Bung : to the Difference between the Bung-Area and the Area of that Circle, *viz.* the Area of the Liquor's Surſace.

Then, from the Bung-Area ſubtract one third Part of the aforeſaid Difference, *viz.* between the Bung-Area and the Area of the Liquor's Surface; multiply the Remainder by the Liquor's Diſtance from the Bung, and the Product will ſhew what Quantity of Liquor is either above or under half the Content of the Caſk.



Example. Let us again ſuppoſe the Caſk in Page 317 whoſe Length is 40 Inches, Bung-Diameter 32, and Head-Diameter 24, and ſuppoſe the wet Inches SH 26 Inches.

The Square of half the Length is 400, the Diſtance of the Liquor's Surface from the Bung SI is 6, whoſe Square is 36; the Area of the Bung-Diameter 2.8519 Ale Gallons, and the Area of the Head-Diameter 1.6042; the Difference 1.2477. Then,

As 400 : 1.2477 : : 36 : .0751.

One Third is = .0250

From 2.8519 Bung-Area,
Subtr. .0250 A Third of the Difference,

Rem. 2.8269

6 Multip. Diſtance from the Bung.

16.9614 Content above the Bung.
And 48.7 Half the Content of the Caſk.

65.66 The Quantity of Liquor in the Caſk.

PROB

PROBLEM XI.

Gaging of MALT.

TO find the Quantity of Malt in a Ciſtern, or upon a Floor.

Firſt, find the Area of the Baſe in Buſhels, by multiplying the Length by the Breadth, and dividing the Product by 2150.42, or only by 2150, and multiply that Area by the mean Depth: (how to take the mean Depth, ſee *Problem* II.) If the Baſe be circular, or oval, divide by 2738. (See *Problem* I.)

Example. There is a Ciſtern whoſe Length is 84 Inches, and Breadth 54 Inches, and the mean Depth is 43.6 Inches; what is the Content?

Multiply 84 by 54, and the Product is 4536; which divided by 2150, and the Quotient is 2.1097 Buſhels, the Area of the Bottom at 1 Inch deep; which multiplied by the Depth 43.6, and the Product is 91.98 Buſhels, the Content.

Example. Suppoſe a Quantity of Malt upon a Floor whoſe Length is 245 Inches, and the Breadth 184 Inches, and the mean Depth 5.6 Inches how many Buſhels are there?

Multiply 245 by 184, and the Product is 45080, which divided by 2150, the Quotient is 20.967, the Area of the Baſe, which multiplied by the mean Depth, the Product is 117.4 Buſhels, the Content.

By the Sliding Rule.

There is an inverted Line of Numbers upon ſome ſliding Rules, marked with the Letter M, which was contrived purpoſely for gaging of Malt; and there is a double Line of Numbers upon the Rule, and upon the Slider two double Lines of Numbers; all theſe are of equal Radius, and all work together at once: Thus ſet the Length and Breath againſt one another upon the inverted Line, and that which ſlides by it; then, on the other Edge of the Rule, againſt the

Depth, you will find the Content in Buſhels: Thus, in the firſt *Example*, ſet 54 upon the Slider againſt 84, upon the inverted Line, and then, againſt 43.6, upon the other Part of the Rule, is 91.98 upon the Slider.

Again, in the ſecond *Example*, ſet 184 upon the Slider to 245 upon the inverted-Line, and againſt 5.6 upon the other Part of the Rule is 117.4 upon the Slider.

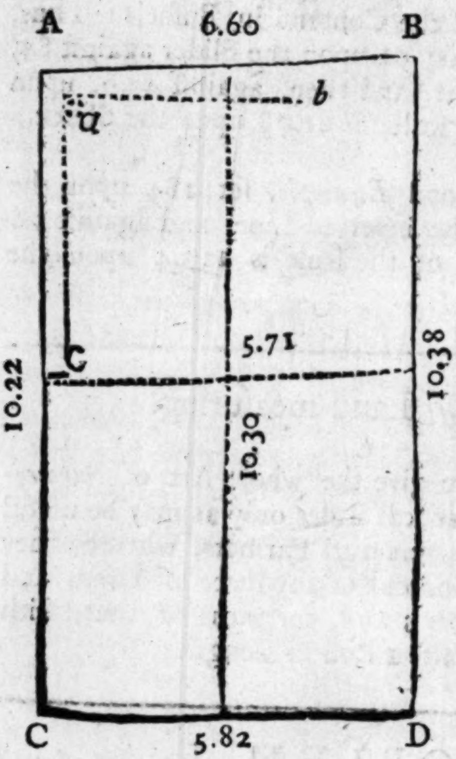
§ II. *Of* Land-meaſuring.

I SHALL not here give the whole Art of *Surveying*, but ſuch practical Rules only as may be uſeful to the Country Graziers and Farmers, whereby they may find the true Content of any Piece of Land, and that by the Chain only; and, for want of that, with a Pole or Stick of half a Rod in Length

PROBLEM I.

To find the Content of a Piece of Land, in the Form of a right-angled Parallelogram, or Long Square, or what is ſomething near that Form.

TO know whether any Angle in a Field be a Right-angle or not, you may take a Piece of Board about 4 or 5 Inches broad, and an Inch thick, either round or ſquare, and with a Saw cut two Kerfs, croſſing each other at Right-angles, and bore a Hole in the middle of the back Side, to put it upon the End of a Stick; this will repreſent the Inſtrument called a Croſs.



Suppoſe you would obſerve the Angle A, to know whether it be a Right-angle, (or near thereunto) prick up your Stick, with the Croſs upon it, a little Diſtance from the Fence, as at *a*, and having ſet up two Marks, as at *b* and *c*, of equal Diſtance from the Fence, turn one of the Slits directed towards *b* ; and then, if the other be directly pointing to *c*, it is a Right-angle.

To meaſure ſuch a Piece of Ground as this Figure above : If you meaſure round, and add the oppoſite Sides together, and take half the Sum, (if they be not equal) or elſe meaſure down about the Middle of the Length and Middle of the Breadth, thus, the Side AB being meaſured, it will be 5.60, that is, 5 Chains and 60 Links ; and the oppoſite Side CD is 5 Chains 82 Links ; the half Sum thereof is 5.71 : And the Side BD is 10.38, and the Side AC 10.22 ; and the half Sum thereof is 10.30 : (it will be the ſame Thing if you meaſure about the Middle of the Breadth) then multiply this mean Length and mean Breadth together, *viz*, 10.30, by 5.71, and the Product is 58.8130 ; which divide by 10, becauſe 10 ſquare Chains is an Acre) by removing the ſeparating Point one Place towards the Left Hand, and it will be 5.88130, that is, 5 Acres and 88130 Parts; which multiplied by 4, and prick off 5 Places, and it will be 3.52520; which 3 towards the Left Hand are 3 Roods; then multiply the decimal Parts by 40, and prick off 5 Places, and it will be 21.00800; which 21 towards the Left Hand are 21 Perches.

	A.	R.	P.
So the whole Content is ——	5	3	21

See the Work.

	A.	R.	P.
5.71			
10.30	5 :	3 :	21
17130			
571			
5.88130			
4			
3.52520			
40			
21.00800			

NOTE, The Chain here made uſe of is 4 Poles, or Rods, in Length; the whole Chain being 100 Links.

But becauſe every Man that may have Occaſion to meaſure a Piece of Land can't procure a Chain, I will therefore ſhew how you may meaſure a Piece of Land only with a Stick of half a Rod in Length, that is, 8 Feet and 3 Inches; but in *Ireland* 'tis 10 Foot 6 Inches; which Stick divide into five equal Parts, ſo will the whole Rod be divided into ten Parts, and will thereby be adapted to decimal Arithmetick.

But becauſe each of thoſe Parts of the Stick are ſomething large, (each Part being 19 Inches and 8 Tenths in *Great-Britain*, but 25 2 Inches in *Ireland*) it will be neceſſary to take your Dimenſions to half of one of thoſe Parts, and then, for that half Part, ſet 5 in the Place of Seconds, thus; ſuppoſe 3 Parts and a half, ſet it down thus, .35. PROB-

PROBLEM II.

LET us ſuppoſe a Field in the Form of a Long Square, whoſe Length is 45 Rods, 5 Parts and a half, and the Breadth 31 Rods, 4 Parts and a half; what is the Content?

Multiply the Length and Breadth together, and divide the Product by 160, (becauſe 160 ſquare Rods are in an Acre) and the Quotient is Acres.

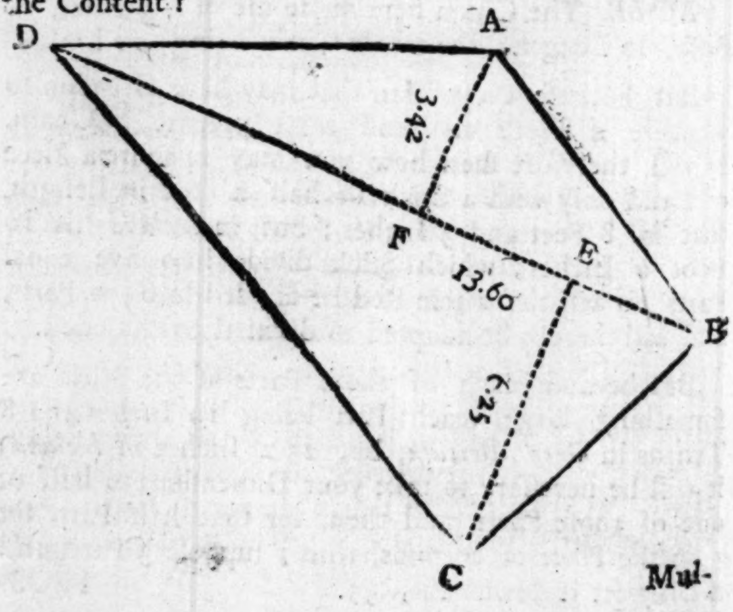
45.55 16|0) 143|2 (8 A. R. P.
31.45 128 *Facit* 8 3 32

22775 4|0) 15|2 (3
18220 12
4555
13665 32

1432.5475

PROBLEM III.

SUPPOSE a Piece of Ground in the Form of a Trapezium, the Diagonal BD 13 Chains 60 Links, the Perpendicular CE 6 Chains 25 Links, and the Perpendicular AF 3 Chains 42 Links; what is the Content?



Multiply the Diagonal by half the Sum of the Perpendiculars. *See* Sect. VI. *of* Chap. I. Part II.

CE=6.25	13.60=BD	
AF=3.42	4.83	
Sum 9.67	4080	A. R. P.
	10880	*Facit* 6 2 11
Half 4.83	5440	
	6.56880	
	4	
	2.27520	
	40	
	11.00800	

By Rods, thus:

CE=25 Rods.	19.34
AF=13.68	54.4=BD
Sum 38.68	7736
	7736
Half 19.34	9670
	16|0) 105|2.096 (6
	96
	4|0) 9|2 (2
	8
	12

A. R. P.
Facit 6 2 12

To

To take the Dimenſions of the Field.

Begin at the Angle B, and meaſure in a direct Line towards D; but when you come at E ſet up your Croſs, and direct one of the Slits to D, and then look through the other Slit, and if it exactly hits the Angle C, then you are juſt in the Place where the Perpendicular will fall; but if it does not exactly hit the Point, move backwards or forwards till it does ſo; then meaſure the Perpendicular, and ſet down the Chains and Links, or the Rods and Parts; then continue your Meaſure towards D; but when you come to F, ſet up your Croſs, and try (as above directed) whether you be in the Place where the Perpendicular will fall: Then meaſure the Perpendicular AF, and ſet down the Chains and Links, or Rods and Parts; then continue your Meaſure to D, and ſet down the Meaſure of the whole Diagonal. This Way of Meaſuring is very exact and true; but the common Way uſed by the Graſiers and Farmers, is to meaſure round the Field, and to take half the Sum of the oppoſite Sides for a mean Side; but the laſt mentioned Piece of Ground being meaſured ſo, will come to

A. R. P. R. P.
7 0 22, which is 2 10 more than the Truth.

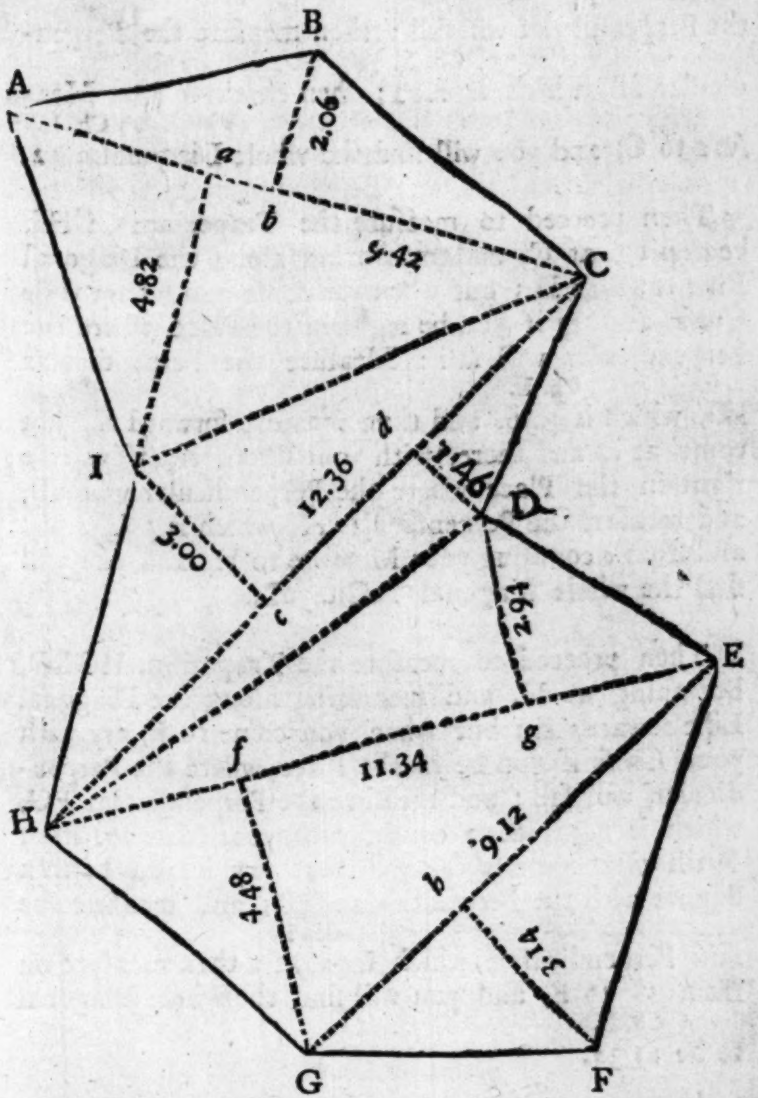
PROBLEM IV.

How to meaſure an irregular Field.

THE Way to meaſure irregular Land, is to divide it into Trapeziums, and Triangles; thus,

Firſt, view over the Field, and ſet up Marks at every Angle, and by thoſe Marks you may ſee where to have a Trapezium, as ABCI in the following Figure.

Then



Then begin and meaſure in a direct Line from A towards C; but when you come to *a* ſet up your Croſs, and try whether you be in a Square to I, (as is before directed) and then meaſure the Perpendicular *a* I, which is 4.82; then meaſure forward again towards C; but when you come to *b*, ſet up your Croſs, and try whether you be in the Place where the Perpendicular will fall; then meaſure the Perpendicular *b*B, which is $\overset{\text{Ch. L.}}{2.06}$; then continue your Meaſure to C, and you will find the whole Diagonal $\overset{\text{Ch. L.}}{9.42}$.

Then proceed to meaſure the Trapezium CDHI, beginning at C, and meaſuring along the Diagonal Line towards H; but when you come at *d* ſet up your Croſs, and try if you be right in the Place where the Perpendicular will fall: Meaſure the Perpendicular *d*D, which is $\overset{\text{Ch. L.}}{1.46}$, and then meaſure forward till you come at *c*, and there, with your Croſs, try if you be right in the Place where the Perpendicular will fall, and meaſure the Perpendicular *c*I, which is 3 Chains; and from *c* continue your Meaſure to H, and you will find the whole Diagonal 12Ch. 36L.

Then proceed to meaſure the Trapezium HGED, beginning at H, and meaſuring along the Diagonal Line towards E; but when you come to F, try with your Croſs if you be in the Place where the Perpendicular will fall; and meaſure the Perpendicular FG, which is 4.48, then continue on your Meaſure from F till you come to G, and there try if you be in a Square with the Perpendicular GD; and meaſure the ſaid Perpendicular, which is $\overset{\text{Ch. L.}}{2.94}$; then meaſure on from G to E, and you will find the whole Diagonal to be $\overset{\text{Ch. L.}}{11.34}$.

Then meaſure the Triangle EFG, beginning at E, and meaſuring along the Baſe EG till you come at *b* and there, with your Croſs, try if you be in the Place where the Perpendicular will fall: and meaſure the Perpendicular *b*F, which is $\overset{\text{Ch. L.}}{3.14}$; continue your Meaſure to G, and you will find the whole Baſe to be 9Ch. 12L. : So have you finiſhed the whole Field.

I have been the larger upon the Explanation of this *Problem,* becauſe moſt Grounds lie in ſuch irregular Forms.

Caſt up the three Trapeziums ſeverally, and alſo the Triangle, and add all the ſeveral Areas together into one Sum, which will be the Area of the whole irregular Plot.

See the WORK

*b*B=2.06	9.42	See Sect. VI. Chap. I
*a*I=4.82	3.44	Part II.
Sum 6.88	3768	
	3768	
Half 3.44	2826	
	3.24048=Area of ABCI.	
*d*D=1.46	12.36	
c I=3.00	2.23	
Sum 4.46	3708	
	2472	
Half 2.23	2472	
	2.75628=Area of CIHD.	
F G=4.48	11.34	
G D=2.94	3.71	
Sum 7.42	1134	
	7938	
Half 3.71	3402	
	4.20714=Area of HGED.	

Baſe=9.12
Half=4.56
Perpend. 3.14

See Sect. V. Chap. I. Part II.

1824
456
1368

1.43184=Area of the Triangle EFG.
3.24048=Area of ABCI.
2.75628=Area of CIHD.
4.20714=Area of HGED.

Sum 11.63574=Area of the Whole.
4

2.54296
40

21.71840

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Facit	11	2	21

Augt. 18th. 1786

FINIS.

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